NEW YORK TIMES BESTSELLER

LOGICOMIX



AN EPIC SEARCH FOR TRUTH

APOSTOLOS DOXIADIS AND CHRISTOS H. PAPADIMITRIOU

ART BY ALECOS PAPADATOS AND ANNIE DI DONNA



A DRAMATIC STORY OF MADNESS AND REASON, LOVE AND WAR

his innovative graphic novel is based on the early life of the brilliant philosopher Bertrand Russell and his impassioned pursuit of truth. Haunted by family secrets and unable to quell his youthful curiosity, Russell became obsessed with a Promethean goal: to establish the logical foundations of all mathematics.

In his agonized search for absolute truth, Russell crosses paths with legendary thinkers like Gottlob Frege, David Hilbert, and Kurt Gödel, and finds a passionate student in the great Ludwig Wittgenstein. But the object of his defining quest continues to loom before him. Through love and hate, peace and war, Russell persists in the dogged mission that threatens to claim both his career and his personal happiness, finally driving him to the brink of insanity.

Logicomin is at the same time a historical novel and an accessible introduction to some of the biggest ideas of mathematics and modern philosophy. With rich characterizations and expressive, atmospheric artwork, it spins the pursuit of these ideas into a captivating tale.

Probing and ingeniously layered, the book throws light on Russell's inner struggles while setting them in the context of the timeless questions he spent his life trying to answer. At its heart, Logicomix is a story about the conflict between an ideal rationality and the unchanging, flawed fabric of reality.

LOGICOMIX

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APOSTOLOS DOXIADIS CHRISTOS H. PAPADIMITRIOU

ART
ALECOS PAPADATOS

COLOR ANNIE DI DONNA

B L O O M S B U R Y

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CONCEPT & STORY

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Alecos Papadatos

COLOR

Annie Di Donna

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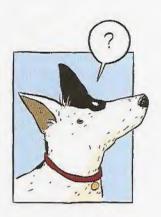
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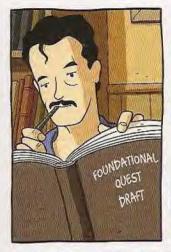
VISUAL RESEARCH & LETTERING
Anne Bardy

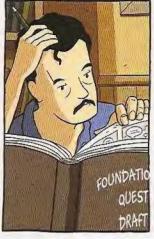
To our children, Eirene, Emma, Isabel, Io, Kimon, Konstantinos, Tatiana, Yorgos

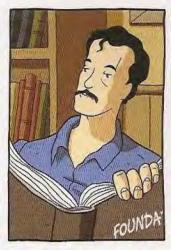
Ύμὲς δ' ἔσεσθε πολλῷ κάρρονες.

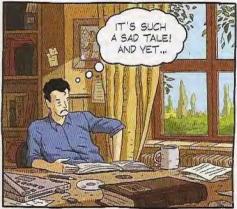
OVERTURE





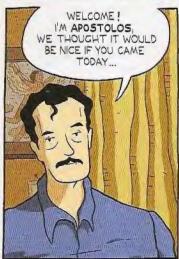




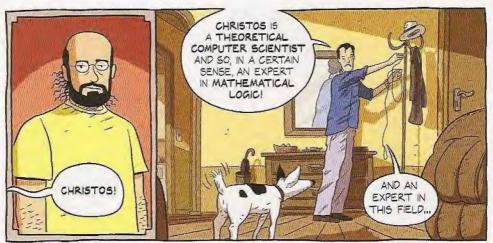






































































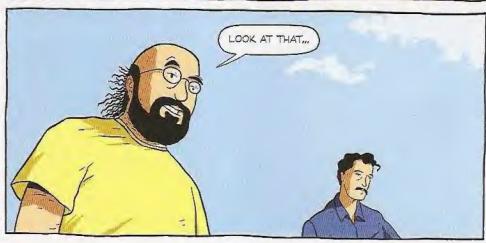




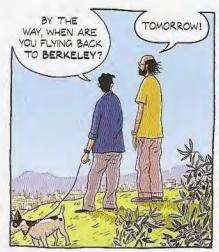






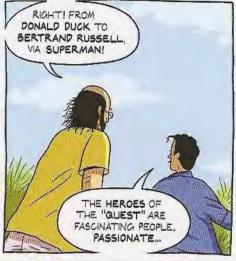




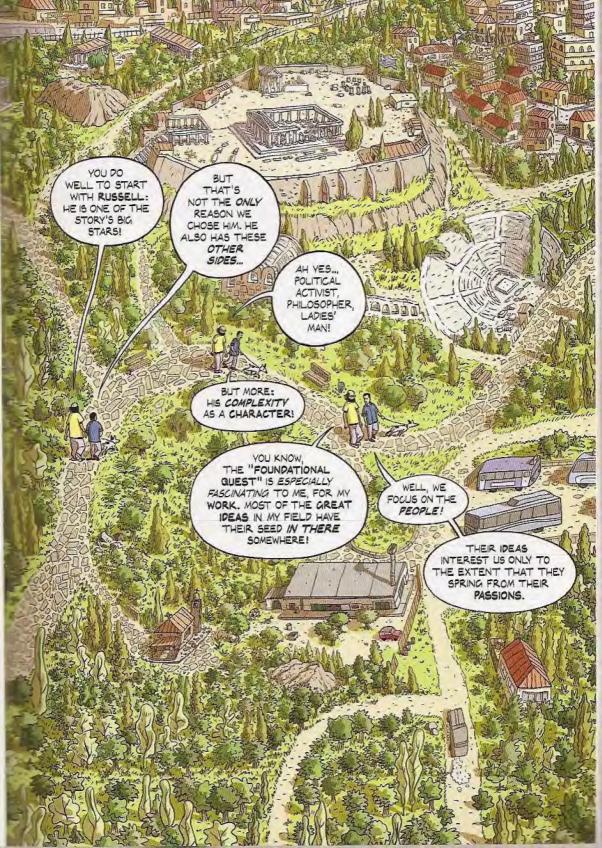


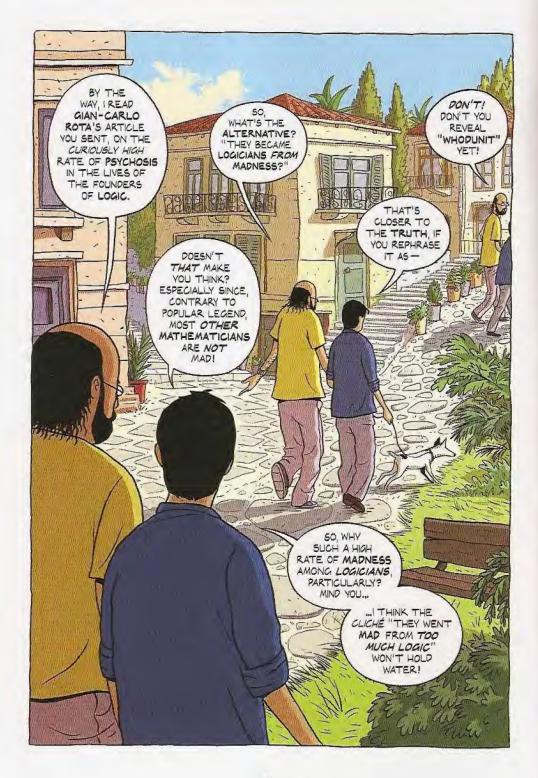


















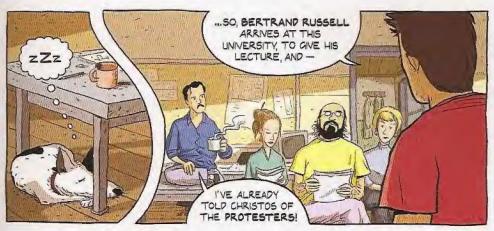


* Apostolos' dog is not named after Japanese comics. "Manga" is a slang word in Greek, meaning something like "cool dude" (U.S.) or "Jack-the-lad" (U.K.).





* Annie is French.









1. PEMBROKE LODGE

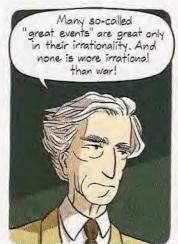


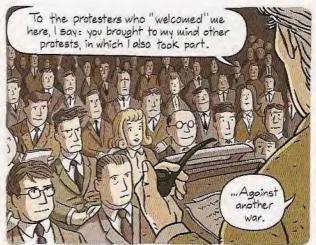


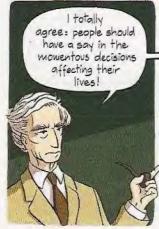


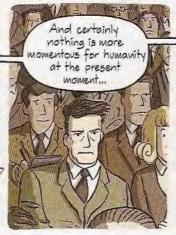
















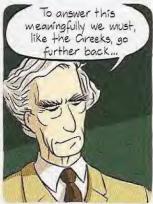


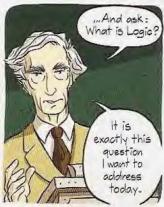






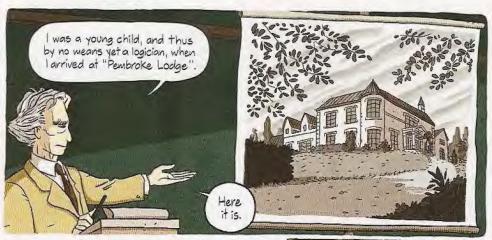
















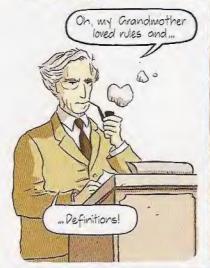






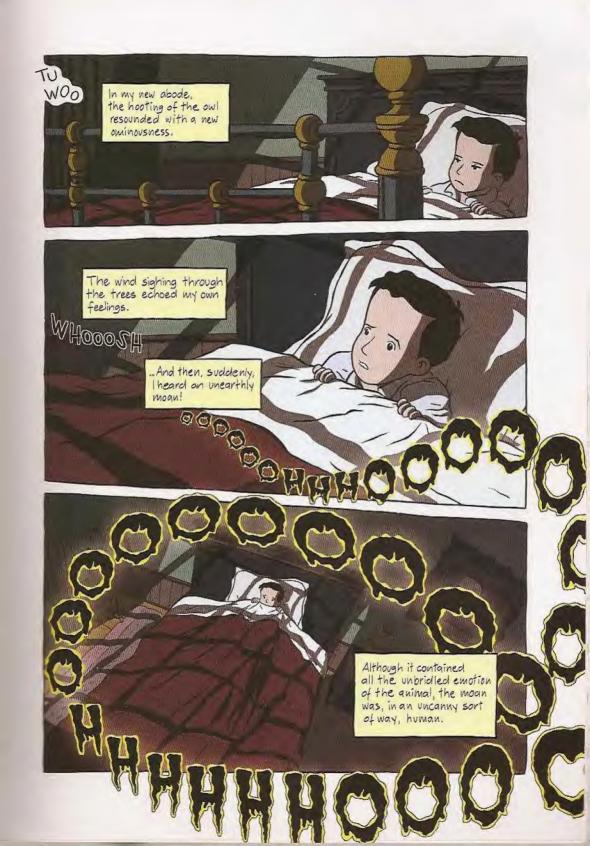














The provenance of this ghastly maan became one of the first mysteries of my life...







Ah, yes! That, of course, was the *greatest* mystery of all!



My father had told we that wy wother had gone on a "very long trip".

So, after he too disappeared, I assumed that he had gone to join her!



...Yet I was receiving such conflicting messages on the matter, it was impossible for me to fathom the truth.



But let me return to the ghostly experience of that first night at the Lodge...









































The tight reins
Grandmother Kept me
under had never given
me the opportunity
for an un-chaperoned
discussion with
Grandpa.







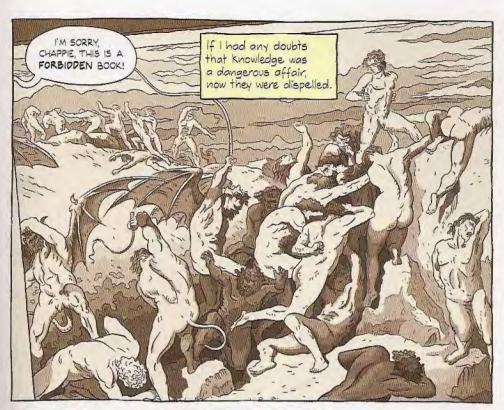




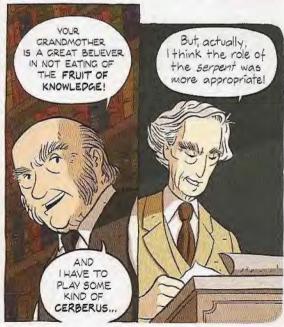


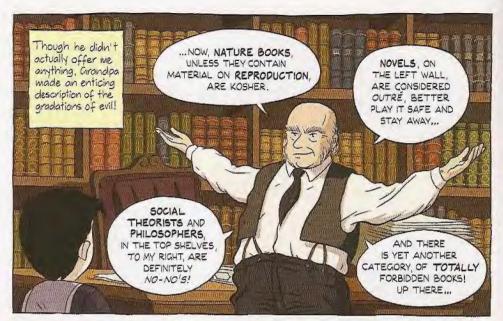














































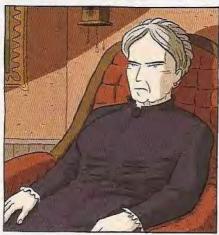




















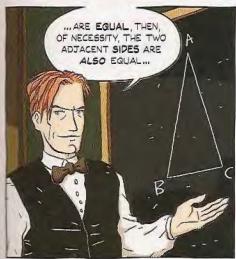




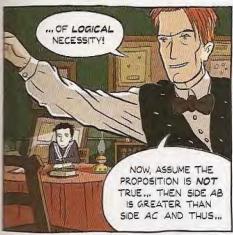


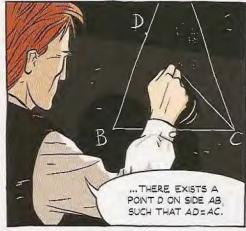






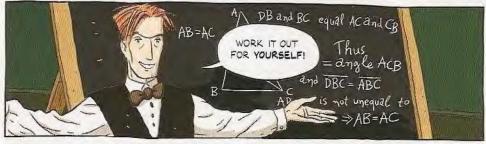




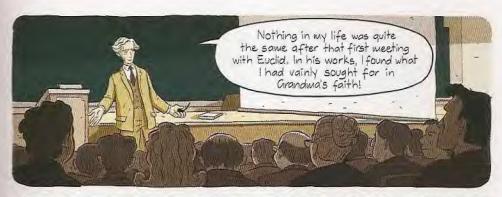


























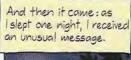






















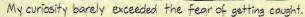




That sawe day, lescaped Grandwother's attention, to follow the route indicated in the mysterious note.

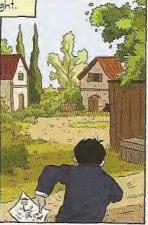














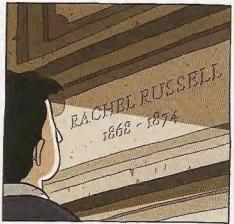


















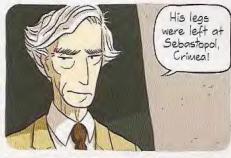








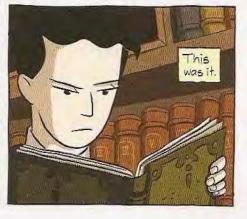








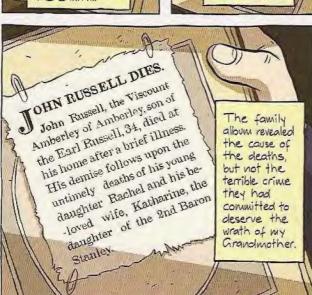






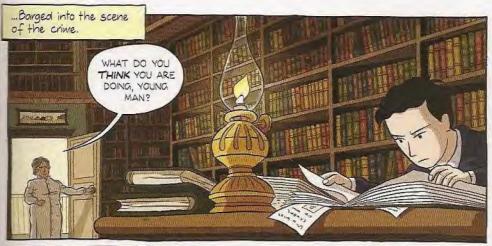


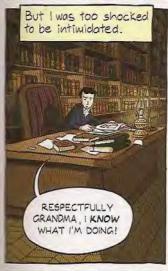










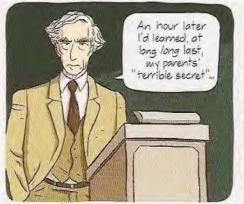








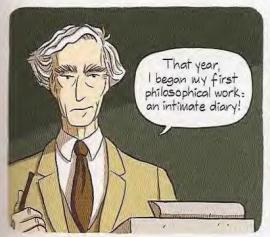


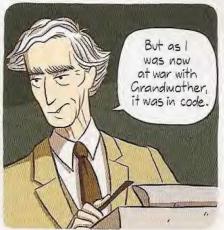


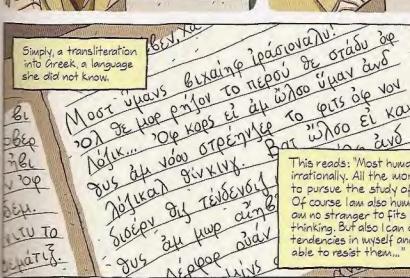




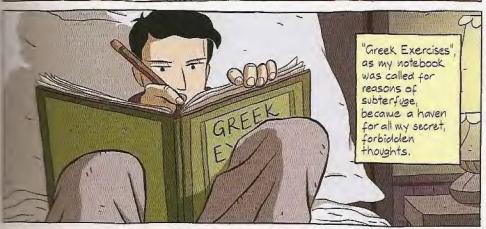


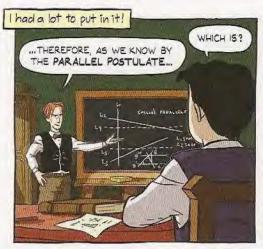


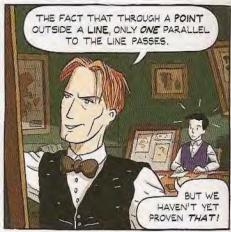




This reads: "Most humans behave irrationally. All the wore reason to pursue the study of Logic ... Of course law also human and thus aw no stranger to fits of non-logical thinking. But also I can discern these tendencies in myself and thus am more able to resist them..."













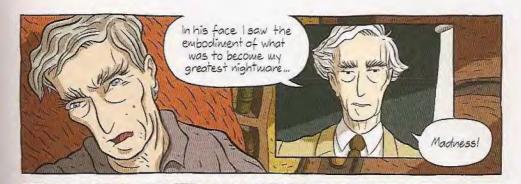


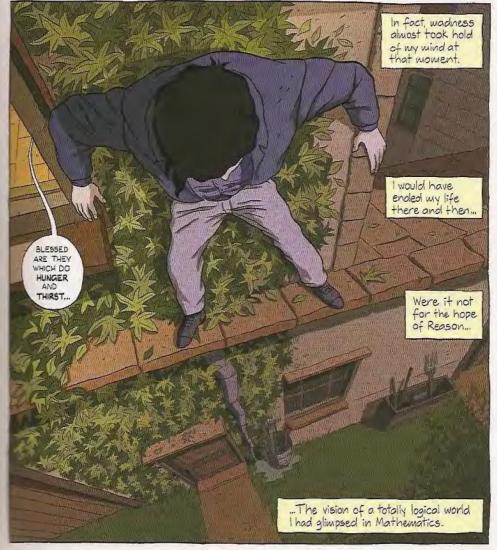










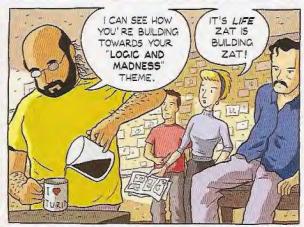


2. THE SORCERER'S APPRENTICE





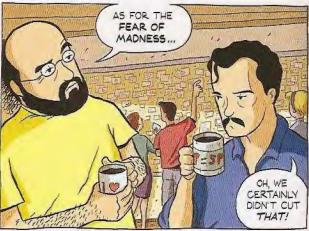


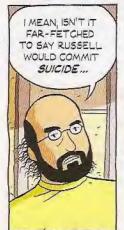




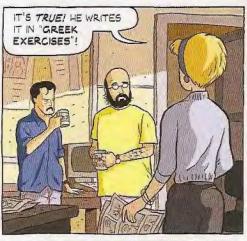


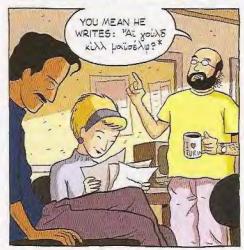






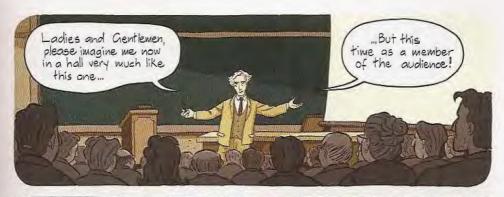








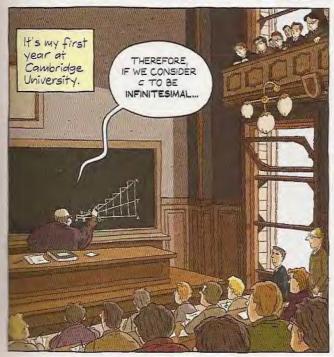
* " | would kill myself."
** "Yes! And now, if you don't mind, the story continues..."



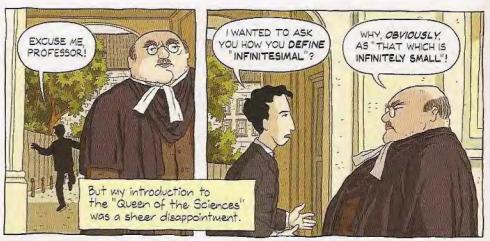




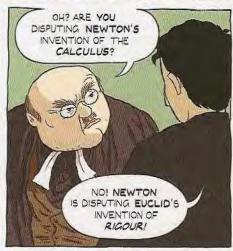


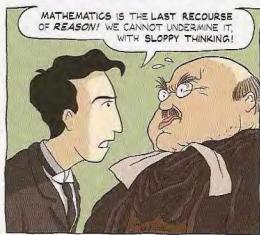










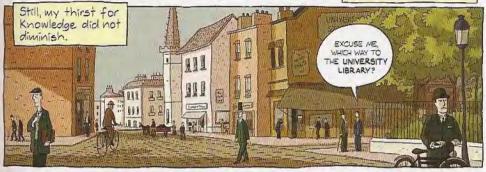


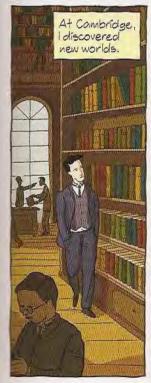


Studying Mathematics I had hoped to penetrate the essence of truth...

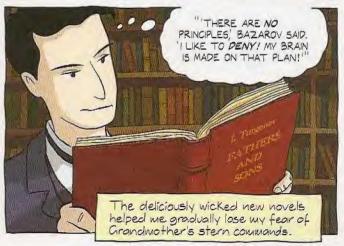


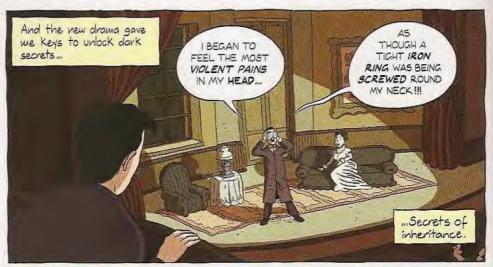












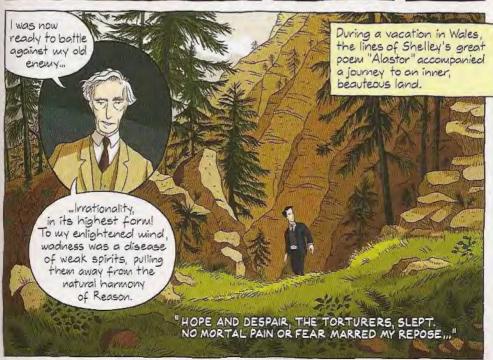










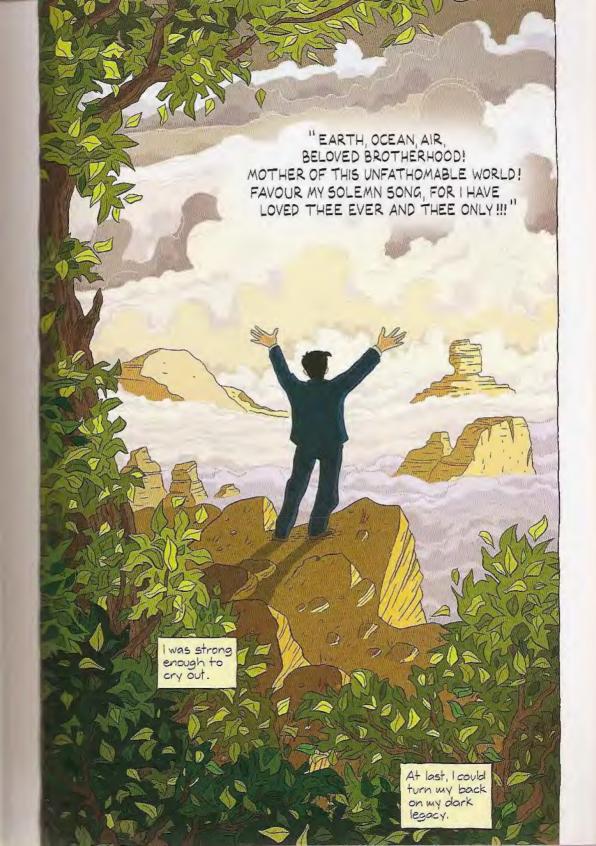














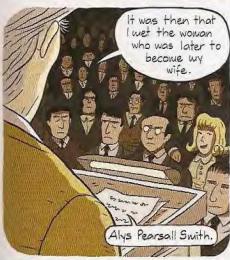




In those years, I was often accompanied by extreme inner tension.









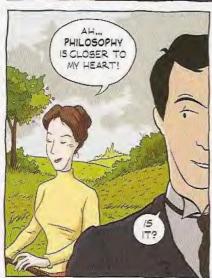








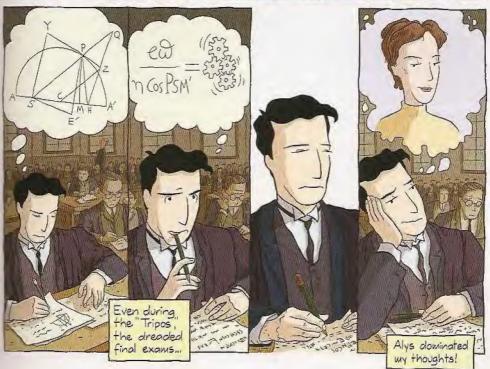


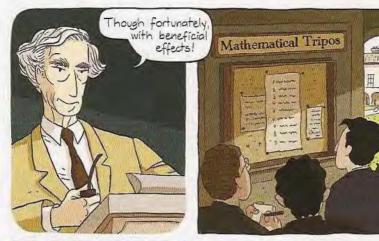




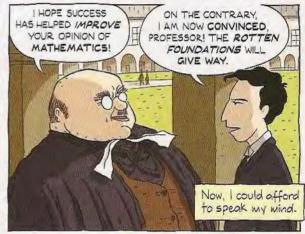








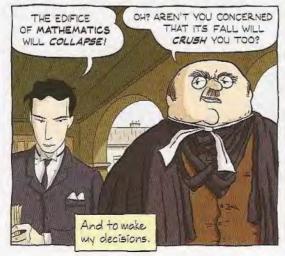




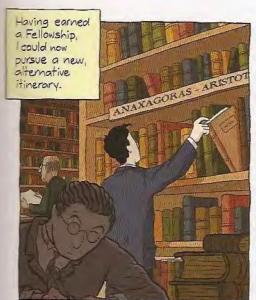
FIRST CLASS

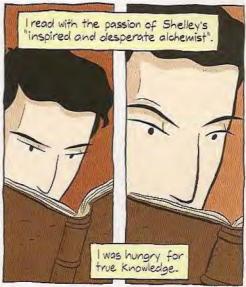
HONOURS ...

NOT BAD!

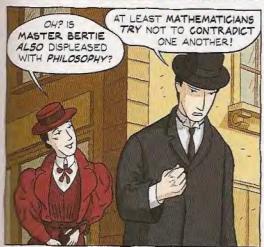


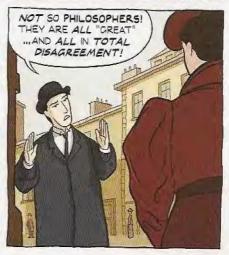


















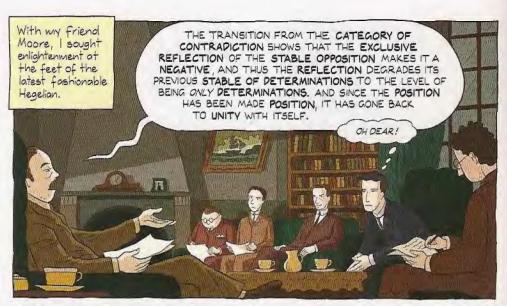






















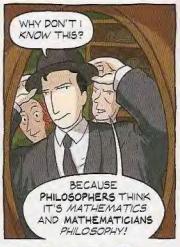






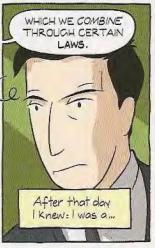










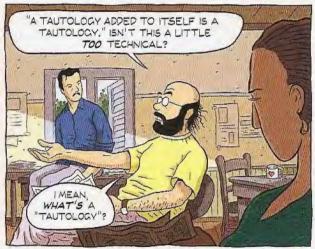


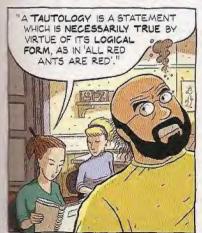








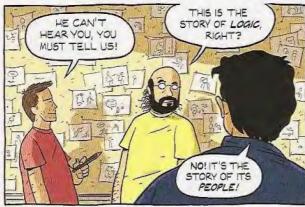




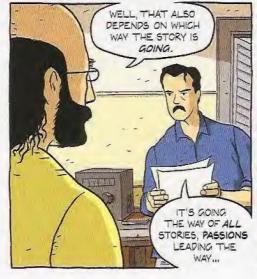














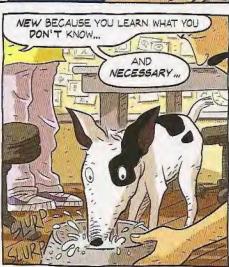








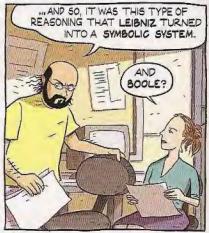


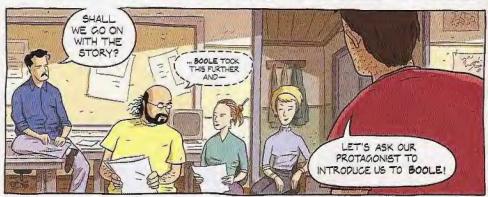




























































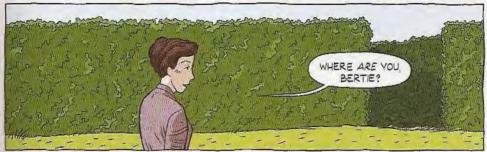








































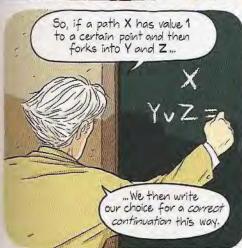


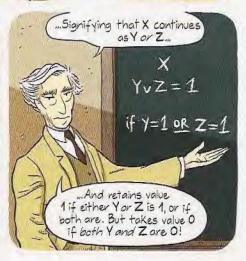


The Hampton Court



To vavigate it, you have to







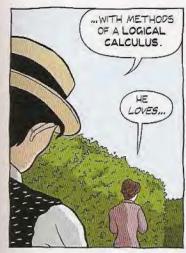




















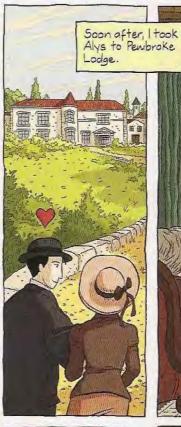










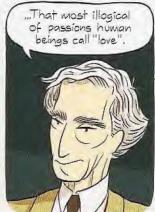


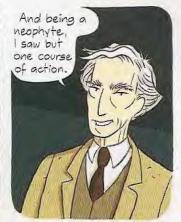








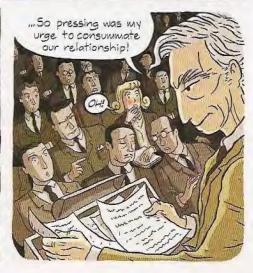


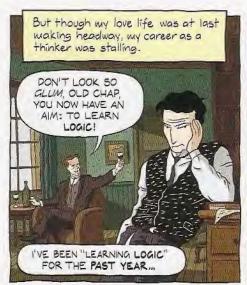








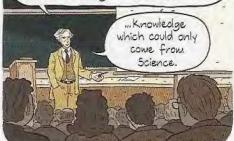




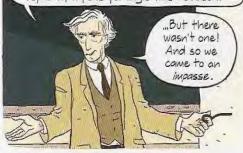


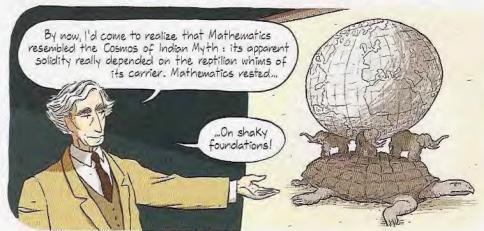


To understand my predicament, remember that my profound, underlying aim had never changed: to acquire certain knowledge about the world...



But Science depended on Mathematics, which was a total wess, plagued by unproven assumptions and circular definitions. To repair it, a powerful Logic was needed...







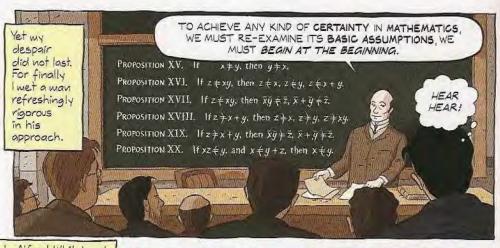


ONLY THEN CAN WE BEGIN TO SET THE HOUSE OF MATHEMATICS IN ORDER!!!



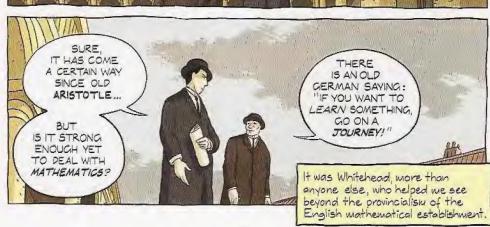


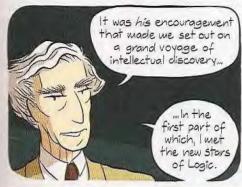




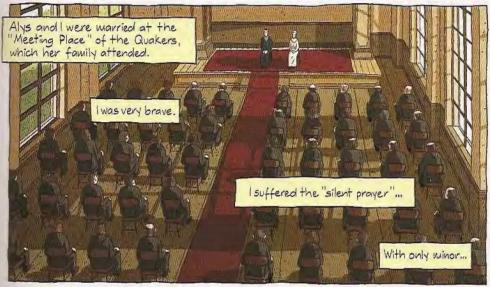
















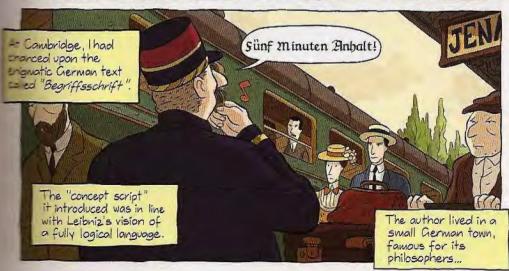


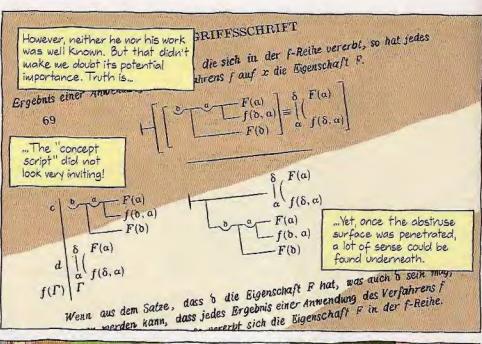
3. WANDERJAHRE













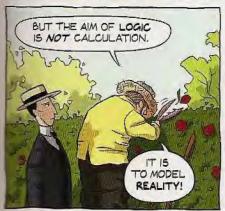














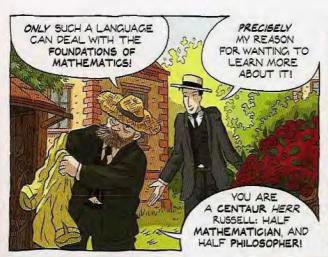




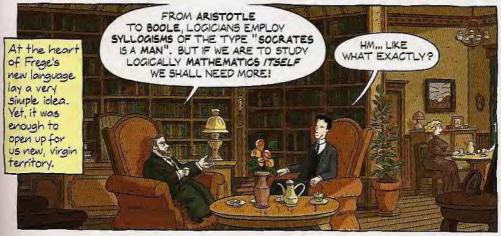


















SHE IS CONSTANTLY LACKING IN UNDERSTANDING

OF MY RIGOUR, HERR DOKTOR,



















BUT WHAT IF HIS















































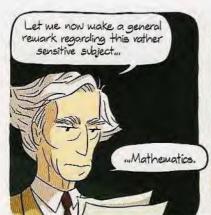


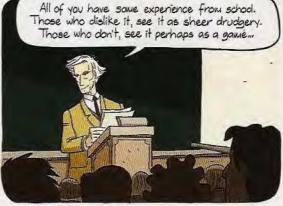






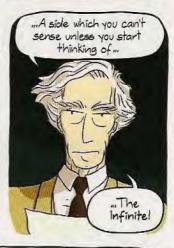






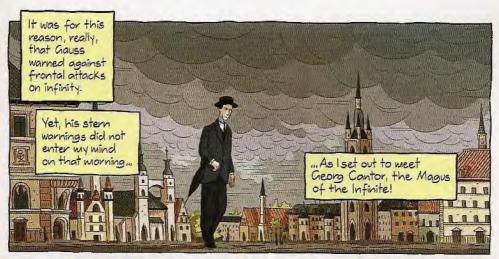










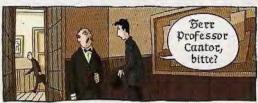




















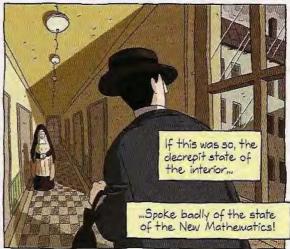










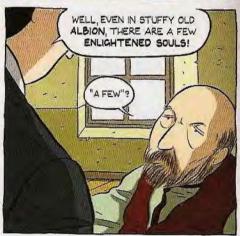








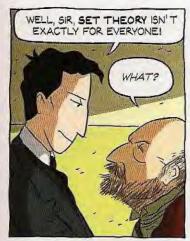




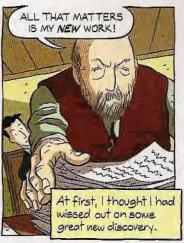


























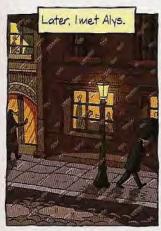
















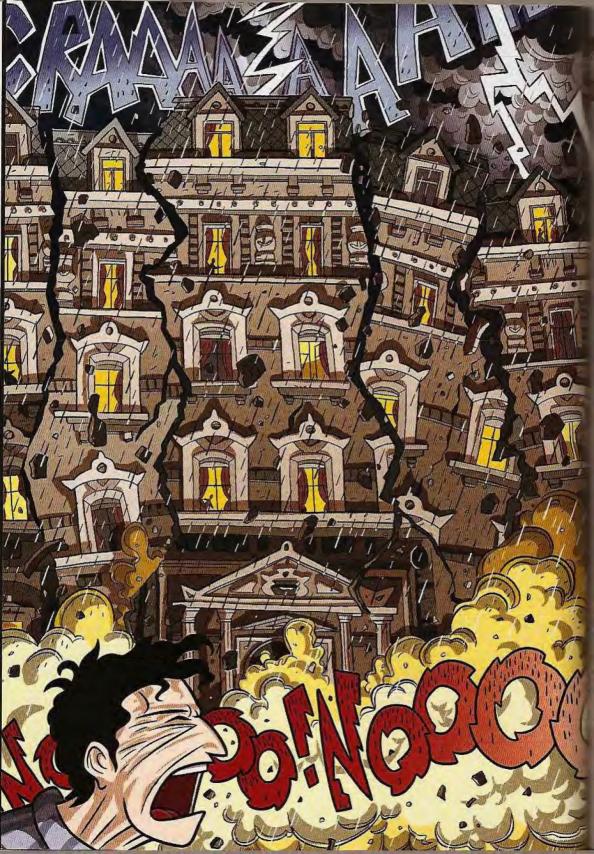








TRAITOR! WHA ... WHAT HAVE I DONE ?! YOU MESSED WITH INFINITY! BUT ... BUT... THAT'S... NOT HÄNDEL, THAT'S... THAT'S ... Gauuuss!!! YOU WRE THE FOUNDATIONS!!!















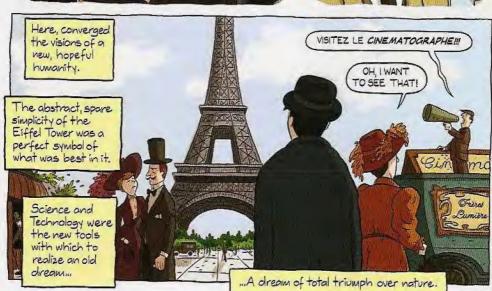


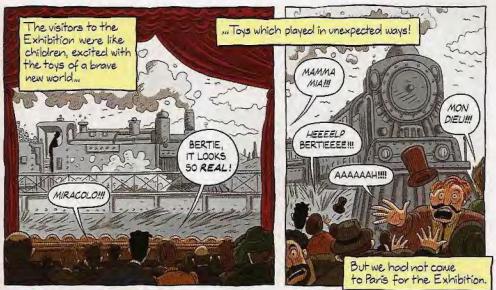




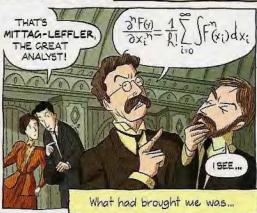
The year was 1900. This was a time of change, a time for new beginnings. And nowhere was this optimism more apparent...

























But I was too excited by new ideas to pay serious attention to new emotions.



And there was no shortage of new ideas: new theories, new techniques, new wethods. There was even ...



Yet my own interest was focused in a single direction...

...The new logical language that would give Mathematics solid foundations. Géométrie Différentielle Salle 1 3° me Étage Salle 13 Rez-de-chaussée Géométrie Hyperbolique Salle 5c 1er Étage this fer Étage de projective terrie Projective fer étrie Projective fer étrie genérales de projective fer étrie projective fer é

It was clear that a central role in this would be played by the Theory of Sets.



...On whose value the Congress's two greatest stars vehewently disagreed!





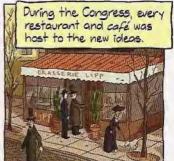
























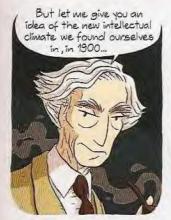










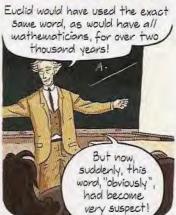


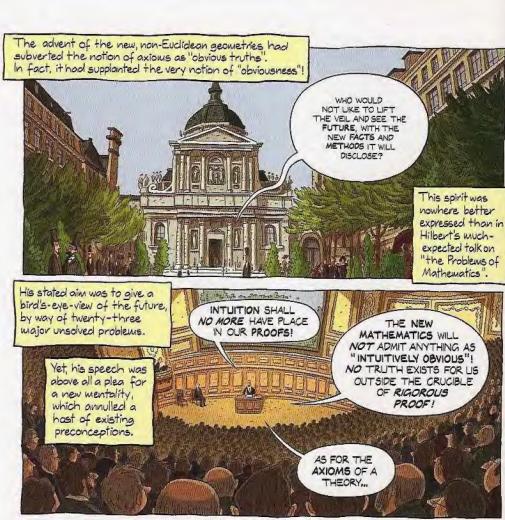


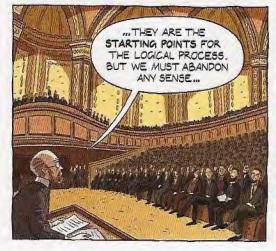


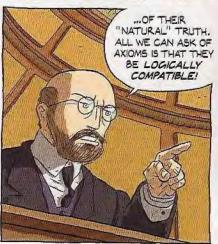






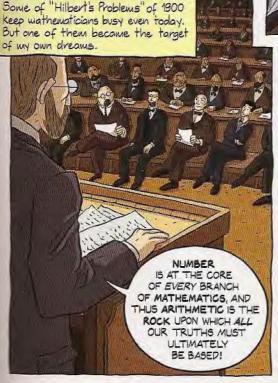








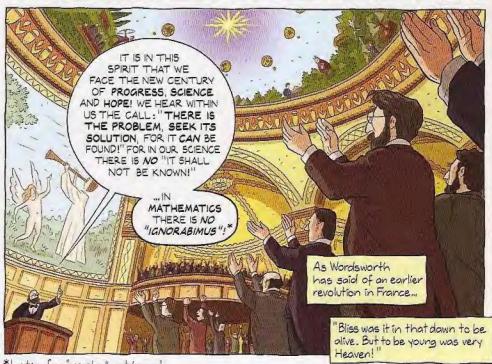












I crossed the Channel with my heart firmly set on the course it was to follow henceforth. But really, I had come full circle, to my first intellectual frustrations.















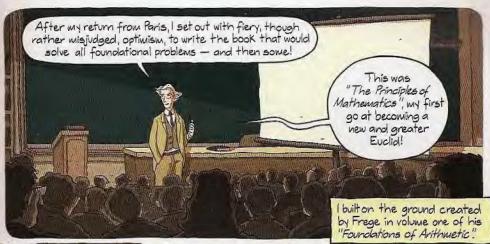


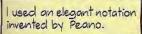




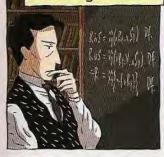
4. PARADOXES

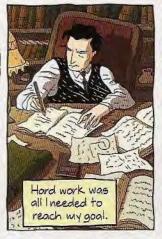






I was convinced I was on the right track.

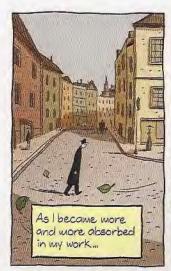








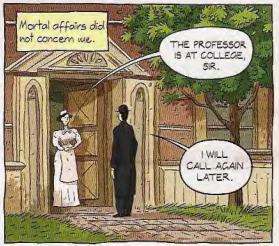
















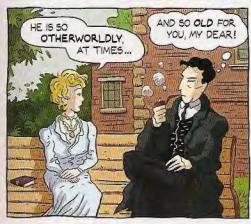


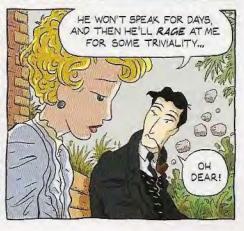


























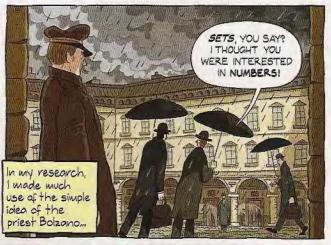
















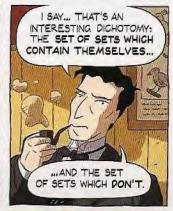


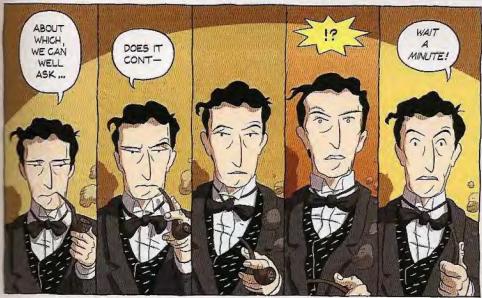


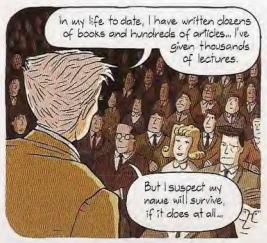










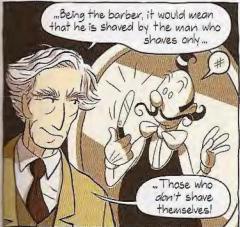










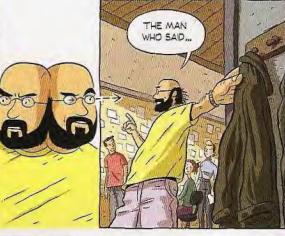


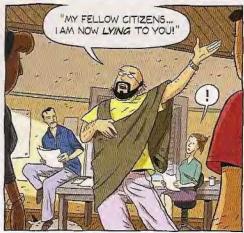












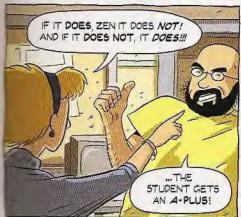






^{*} Of course LOGICOMIX is also self-referential.





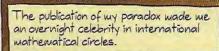






It sounds like a parlor witticism. But it subverts the notion of "set" as a collection defined by a common property...





Some greeted it with joy...

...Like Poincaré, who saw in the paradox strong arguments against any attempt to create purely logical foundations for Mathematics.

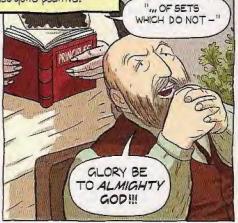
His oft-repeated credo that "Logic is barren" now found a perfect justification...

CONTRADICTIONS!

HA,HA!
THIS RUSSELL
HIT TWO BIRDS
WITH ONE STONE:
LOGIC AND SET
THEORY ARE BOTH
DESTROYED!

ACTUALLY, IT'S NOT BARREN: IT BREEDS































In the end, he did publish volume two of the "Foundations of Arithmetic". But with an addendum.

> Of all the acts of intellectual honesty I have witnessed in my life, none compares with Gottlob Frege's reaction to my paradox.

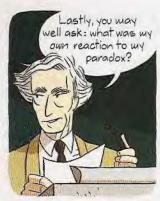
There cannot be greater intellectual courage than this ...

Hardly anything more unfortunate can befall a scientific writer, than to have one of the foundations of his edifice shaken after the work is finished. I was placed in this position by a letter of Mr. Bertrand Russell, just when the printing of this volume was nearing its completion.

DDENDUM

The collapse of one of my laws, to which Mr. Russell's paradox leads, seems to undermine not only the foundations of my Arithmetic but the only possible foundations of Arithmetic as such.

... To put the Truth above all else.





















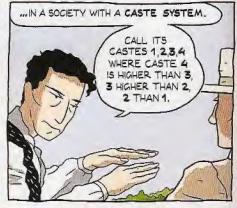








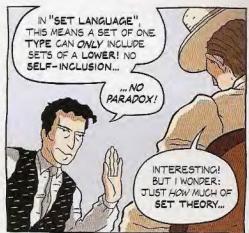
















































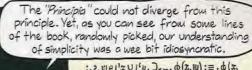


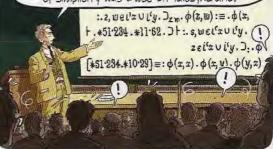
















was as old as

Euclid: a new

set of axious!

Once again, we rebuilt from the bottom.











































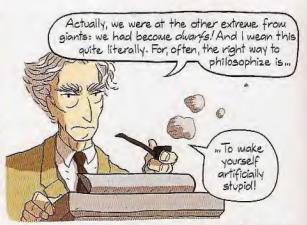










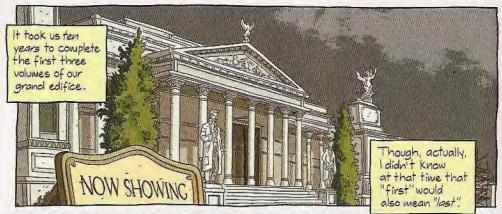








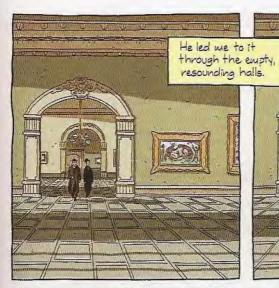






















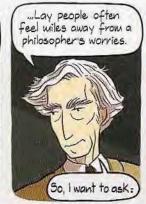




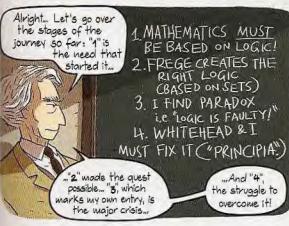


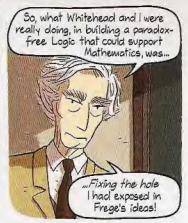








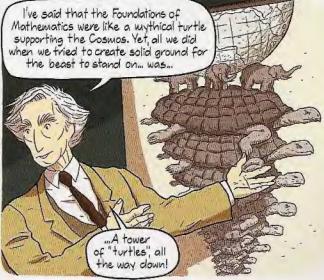






And we were successful

in our task, in all ways but





Yet, despite wy initial reticence to publish, leventually agreed: Maybe a book would help us find new associates in our efforts!



Also, of course, I suffered from a bad case of intellectual cabin fever...





































































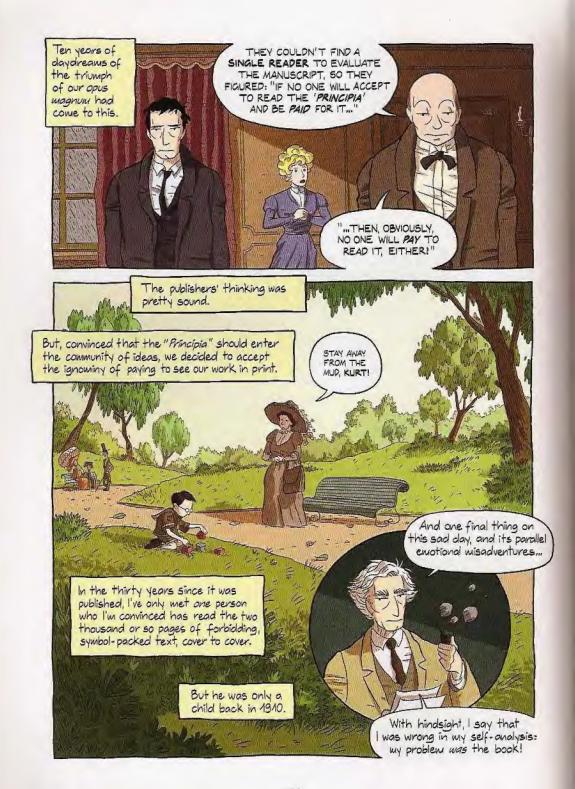










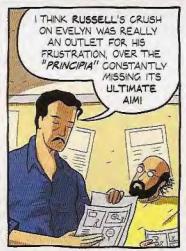


ENTRACTE



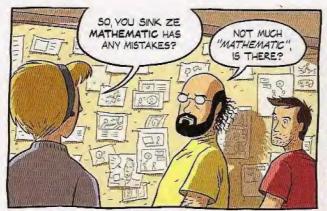


























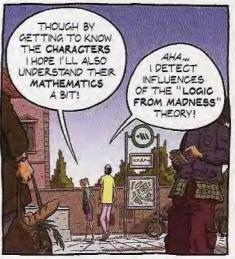














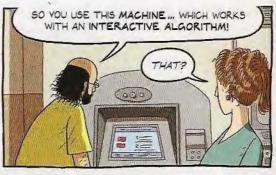




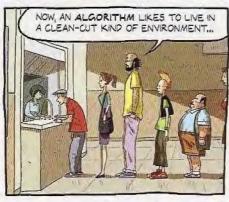




























Hello Friend, I arrived at Berkeley this worning... But I've been thinking of Athens. Of the "Foundational Quest in Comics"... ... To which I've given the nickname "Logicomix" And its meaning. So I want totall you a little story. Which I think brought me a bit closer... "To the "Logic and Madness" theme.







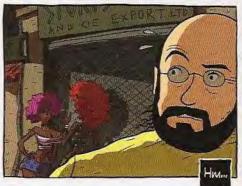












Sure enough, there had been some changes in the decades of my absence!











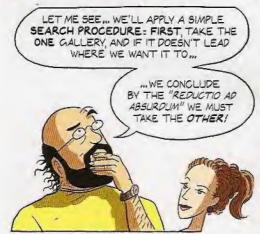
















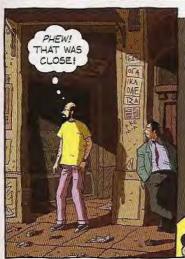




























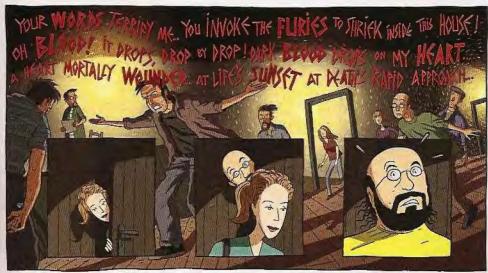
















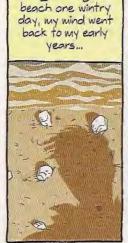


5. LOGICO-PHILOSOPHICAL WARS









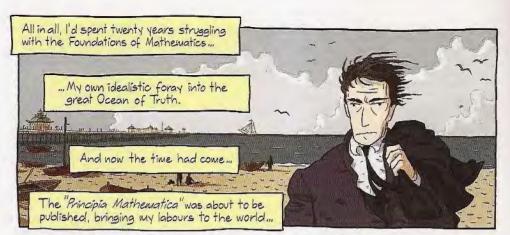
Sitting on Brighton



..The time when











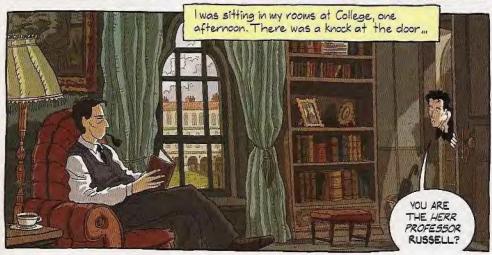




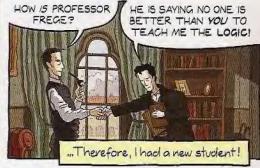
50! We are now in the year 1911. In it, two events of momentous import occurred...

One started as a dreawcome-true but eventually turned into a nightware. The other, on the contrary—









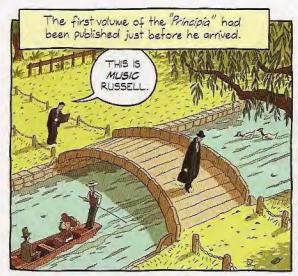
























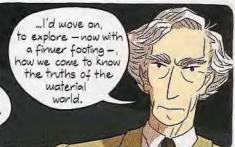






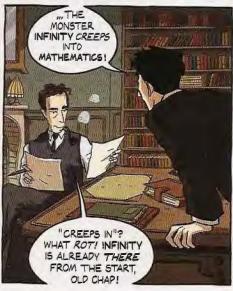


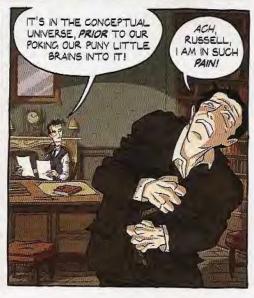


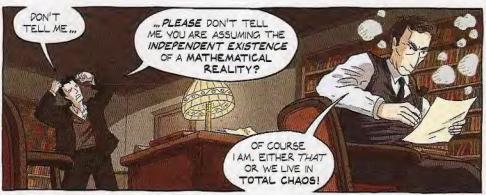






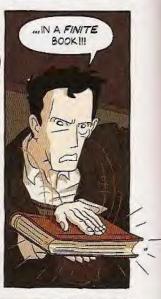










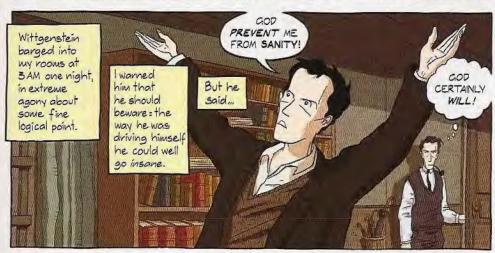










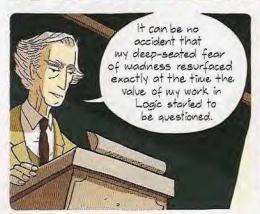


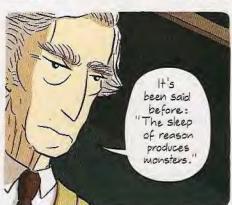
HM. HERE RUSSELL SEEMS TO IMPLY THAT MADNESS COMES FROM LOGIC AND NOT THE OTHER WAY ROUND, AS YOU SAY!

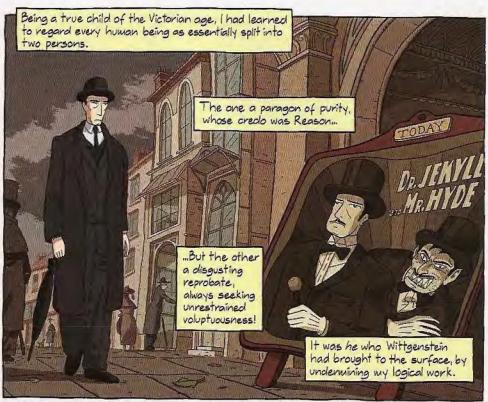












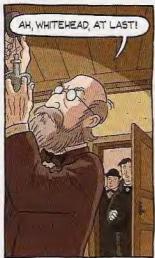




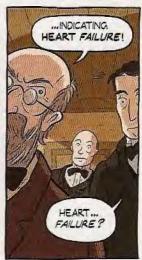














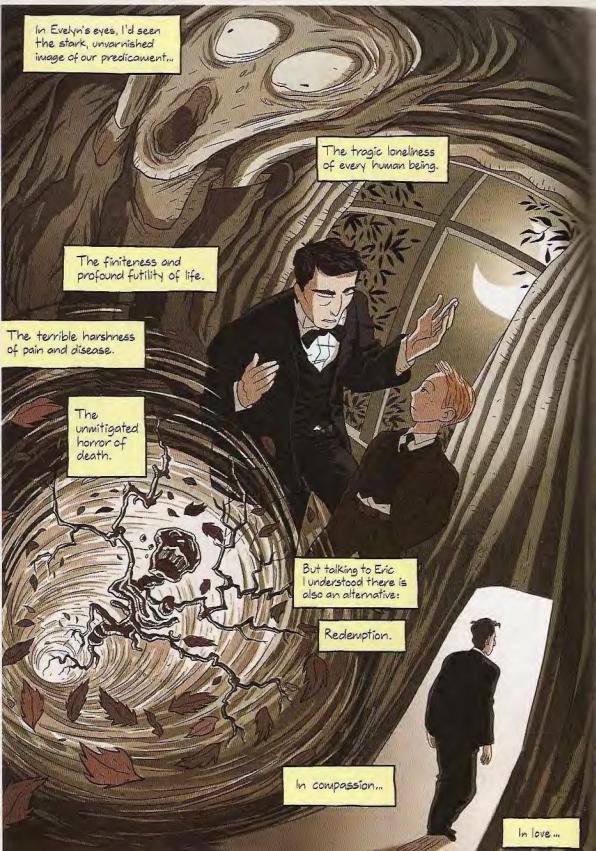








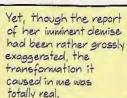












So real, in fact that when I received a letter from Wittgenstein, who had gone to a God-forsaken Norwegian fjord "to think about the wearing of logical propositions"...

... I was not so affected by his doubts, or so perturbed by his criticisms.

Theory of Types Can't me from this of the first of the fi





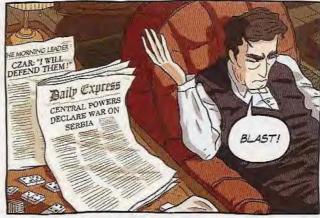


















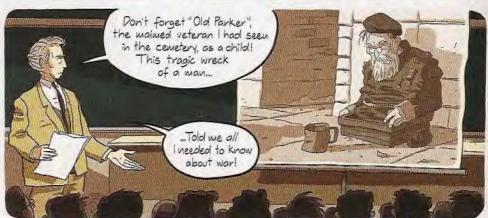








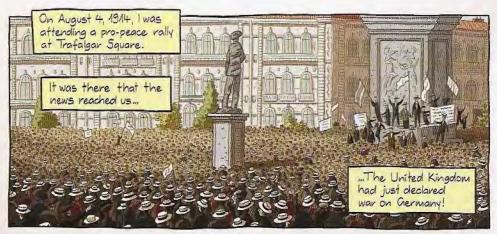


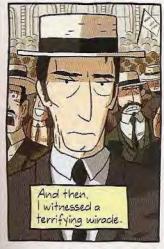


















Thankfully, this strange upsurge of my deep-buried, tribal instincts lasted but a few hours. Then reason took over again.



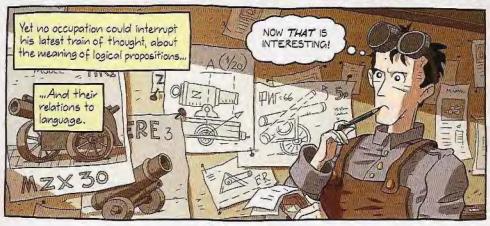
I started, in my lectures and articles, to argue against the madness that was engulfing some of the cleverest people I Knew, including...



My own"Crown Prince of Logic" had now enlisted, as a *volunteer* in the Austro-Hungarian luperial











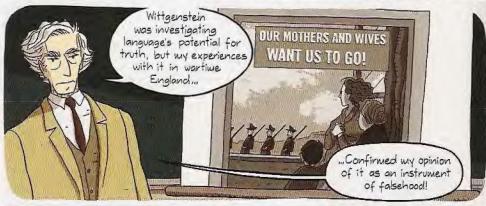






























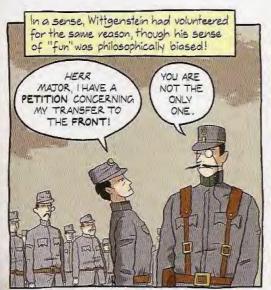






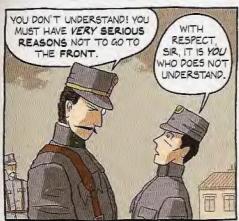


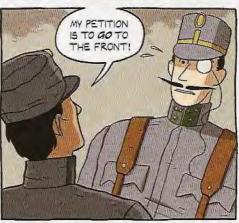


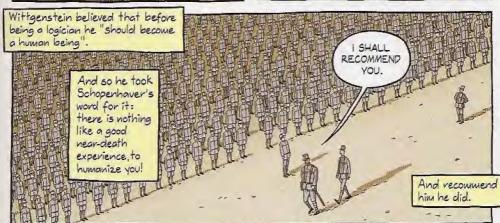


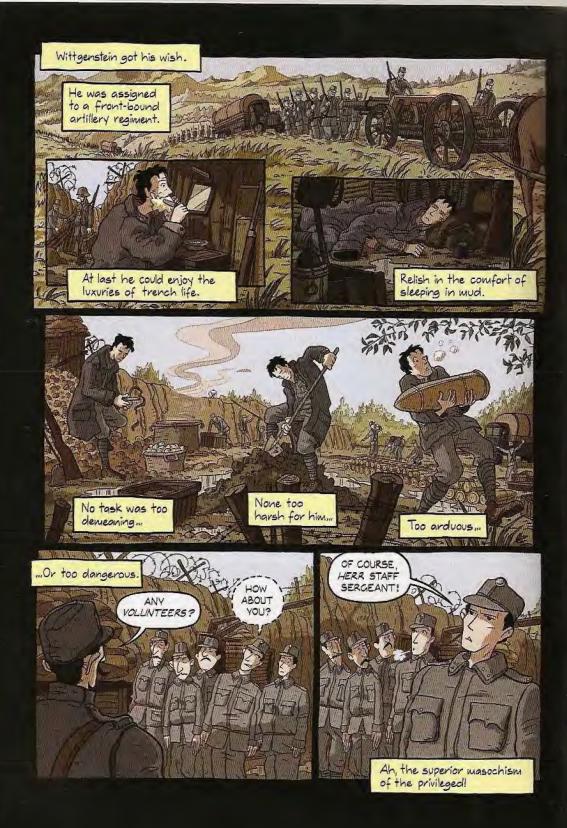


























of the major disadvantages of Reality: from up close, it looks very different from any "picture".





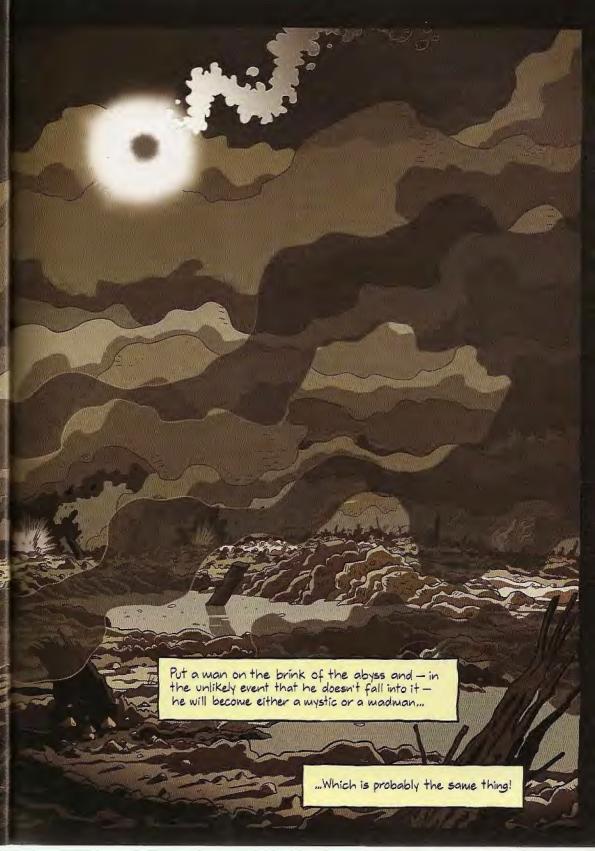








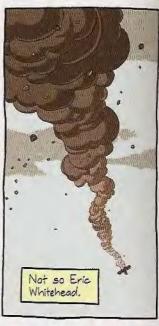


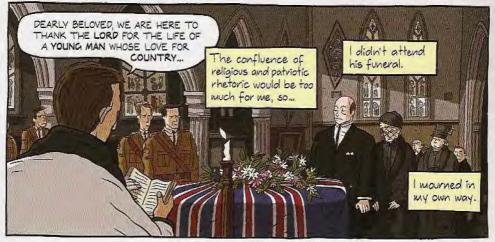




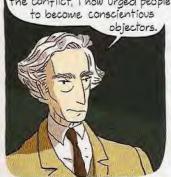






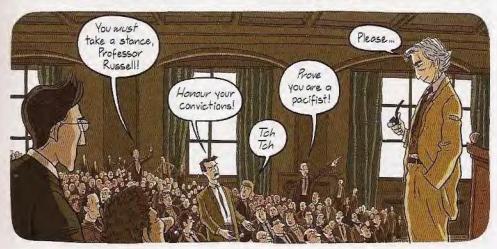


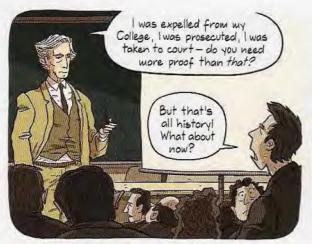
...Which involved changing my tack: from arguing simply for a peaceful resolution of the conflict, I now urged people to become conscientious

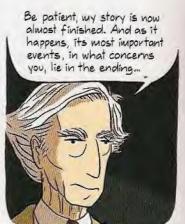










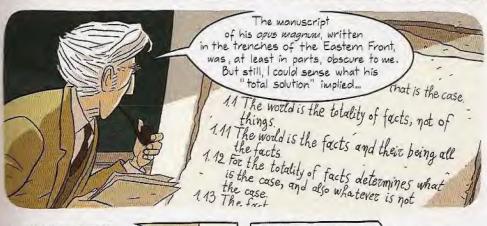










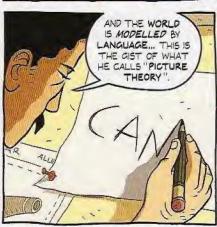












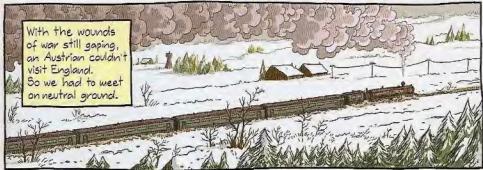


















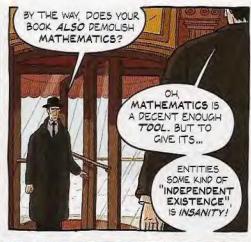




















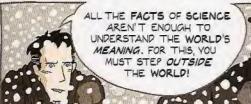












WITHOUT
LANGUAGE
OR THOUGHT,
HOW CAN YOU
UNDERSTAND
ANYTHING?





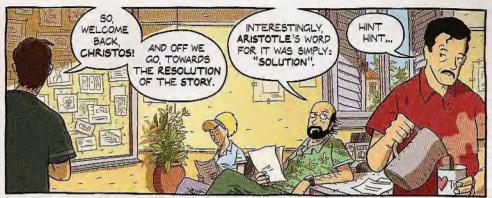






6. INCOMPLETENESS



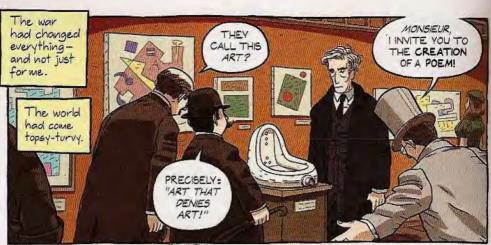






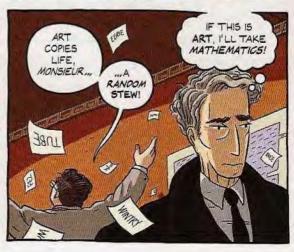






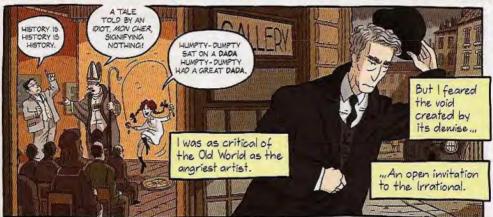






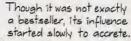




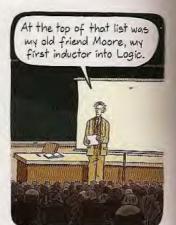


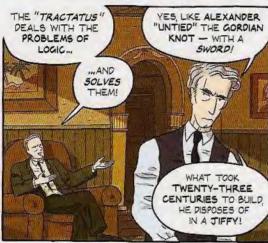


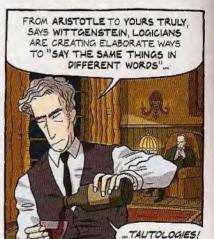












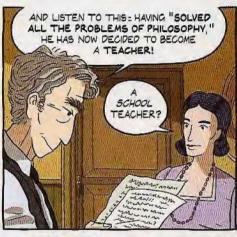




















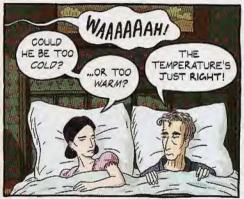






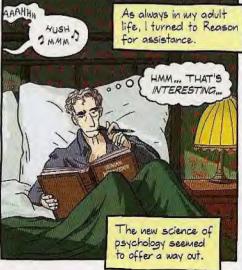








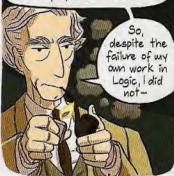




In fact, the time seemed propitious for an extension of my logicist project.



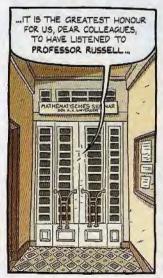
... A project to apply the tools of Logic, Mathematics and the Physical Sciences to the study of human matters.

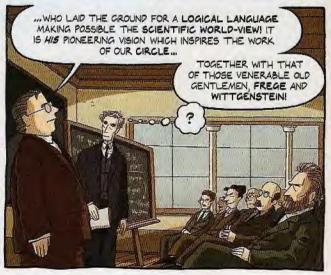














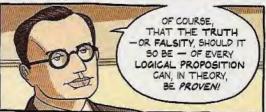




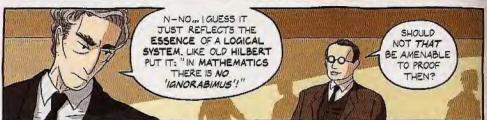


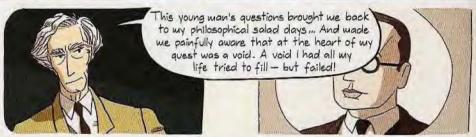
















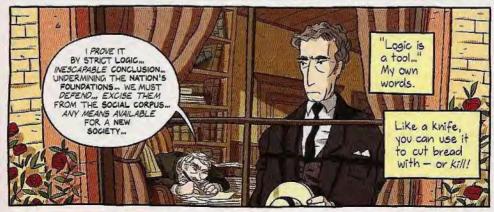












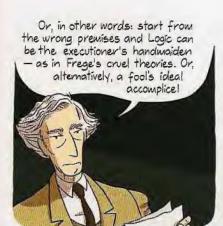


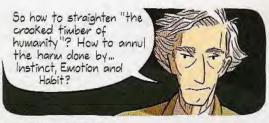


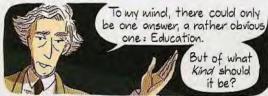










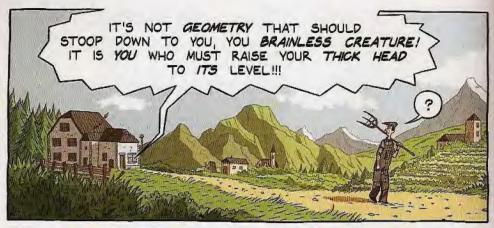






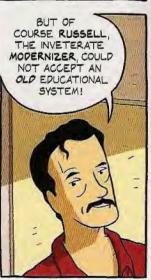








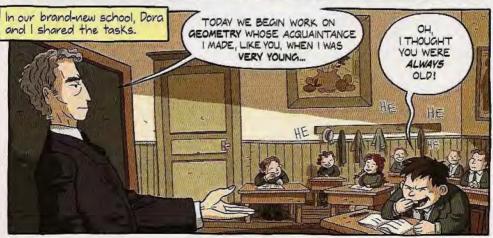






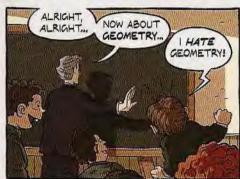


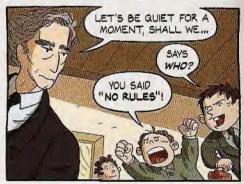












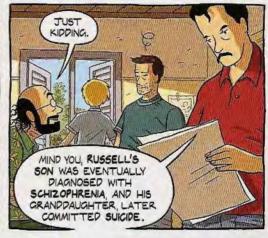








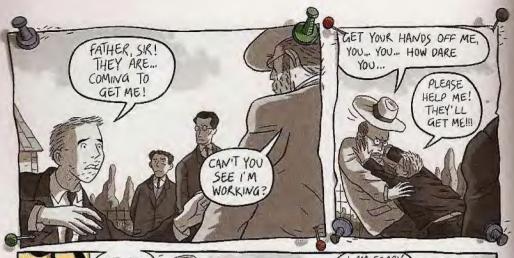








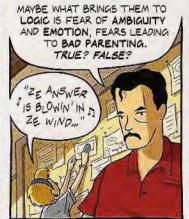
*Zey are crazy zese logicians!

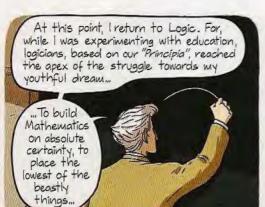




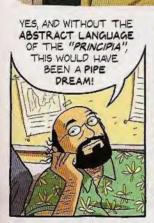


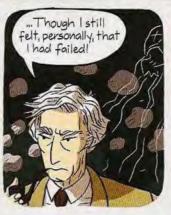


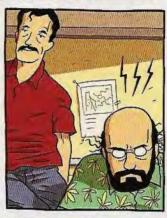


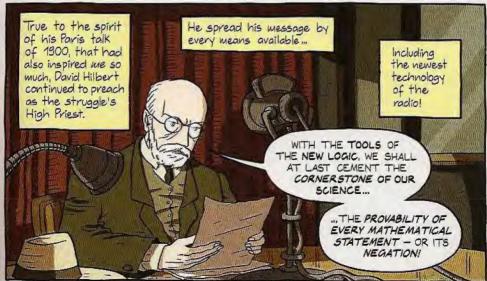




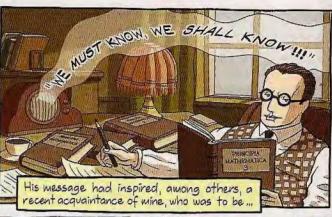






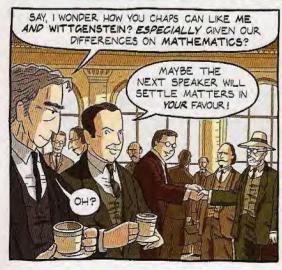


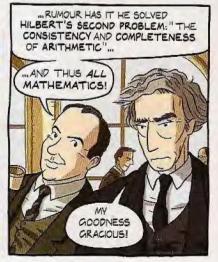




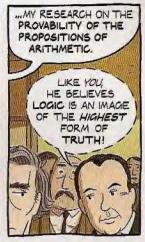


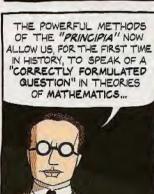


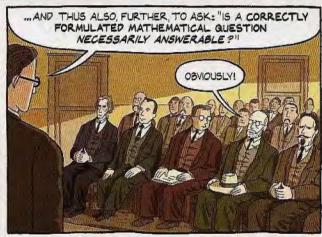


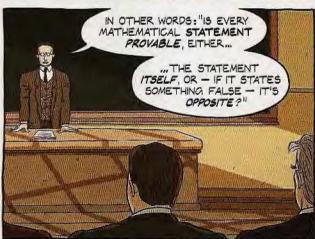
















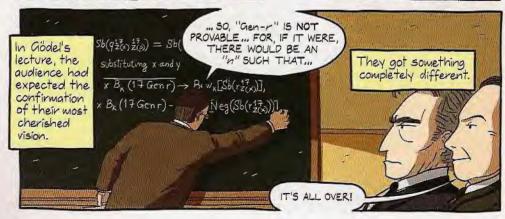




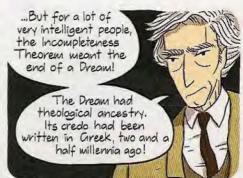








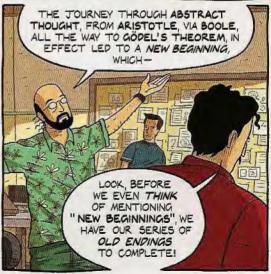














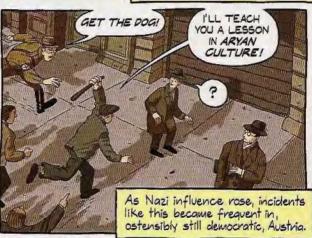


















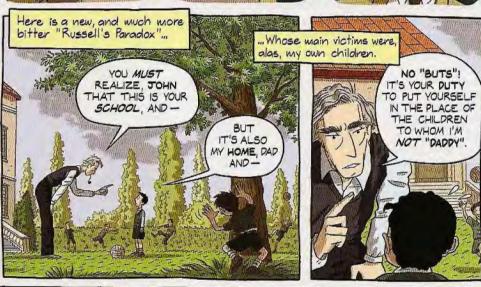


I want to express my gratitude to you, Ladies and Gentlemen, for your company on this, rather long, journey!

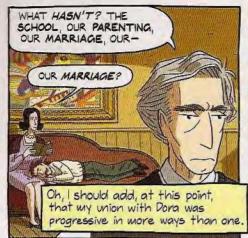


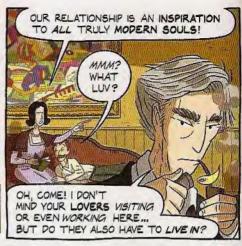
This journey from my

A journey of some joys and more disappointments, the latest of which is the realization that I've failed — also — as an educational reformer.





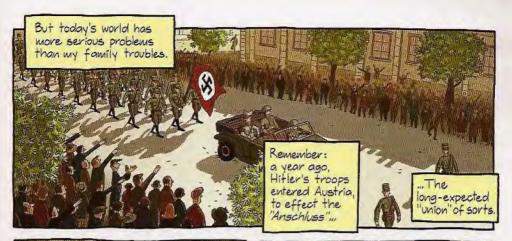




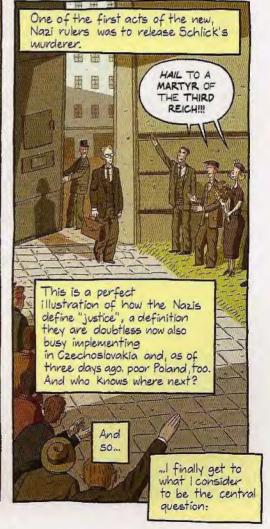




















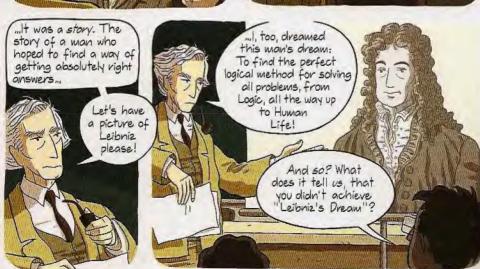
















Directly about the war... maybe nothing. But it tells you a lot about your stance on it. Or, rather about your conviction that you are absolutely right in your views!





Wait! I don't want you to wisunderstand me: even today, I'd define myself as a rationalist! Even now, I believe that Logic is a most powerful tool ...

> ... As far as it goes.

When it comes to talking about human life, it certainly isn't! And when Logic congeals into all-encompassing and perfect-seeming theories, then it can actually become a very evil con trick!



Wittgenstein has a point, you see: "All the facts of science are not enough to understand the world's weaning!

far!



But hear this too: like wany in this hall, still try, and very hard, to remain a pacifist. Yet...



of Hitler and Stalin taking over Europe is too hard to bear!





Listen: take my story as a cautionary tale, a narrative argument against ready-wade solutions. It tells you that applying formulas is not good enough—not, that is, when you're faced with really hard problems!

Russell,
you are still evading
the central question:
"Why should we join
England's war?"



But, you see, there is no dileuma, Professor. It's clear: we shouldn't fight a war that does not directly involve us!







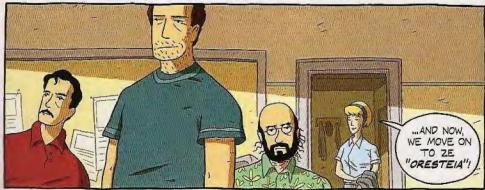












FINALE











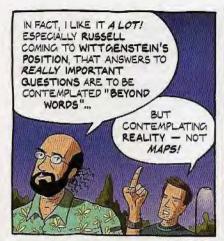










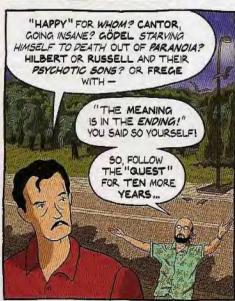




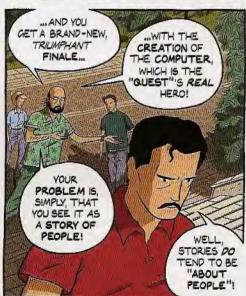






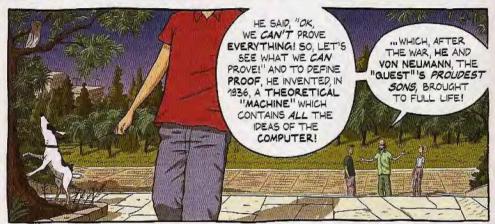










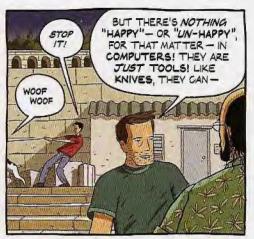




















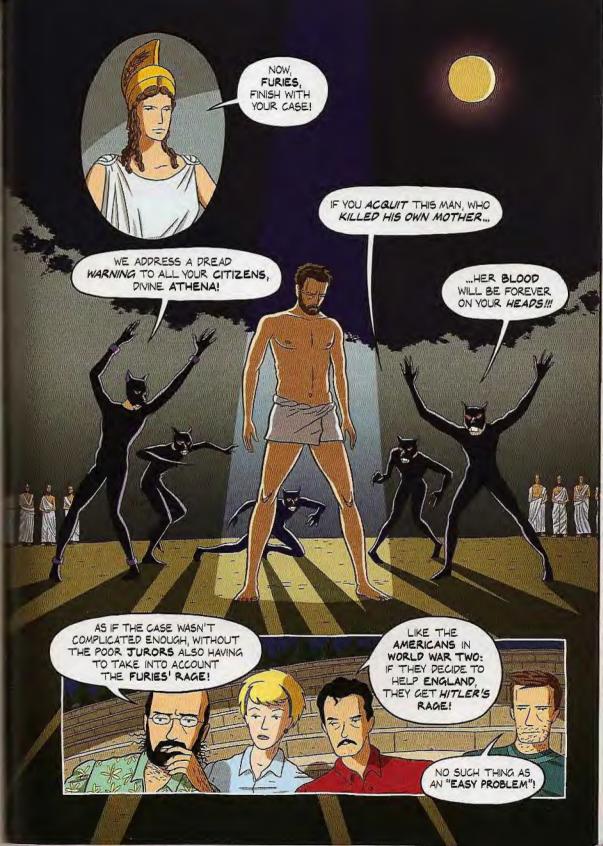


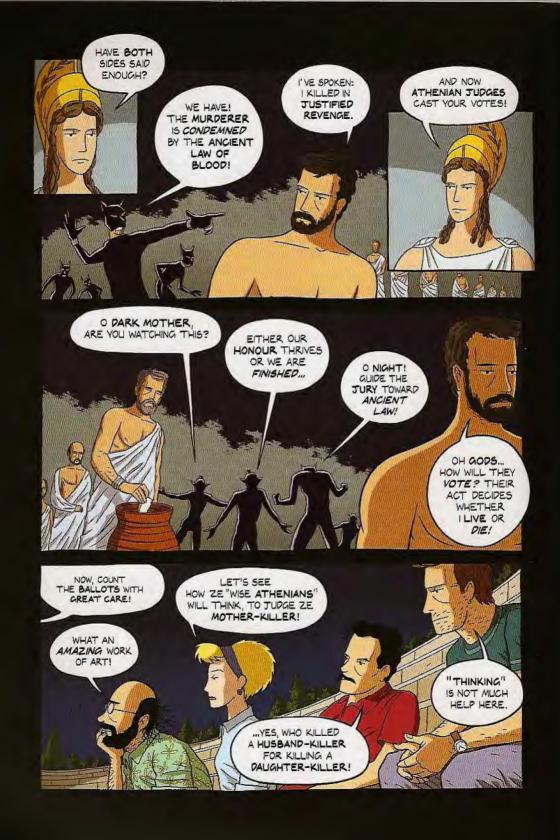






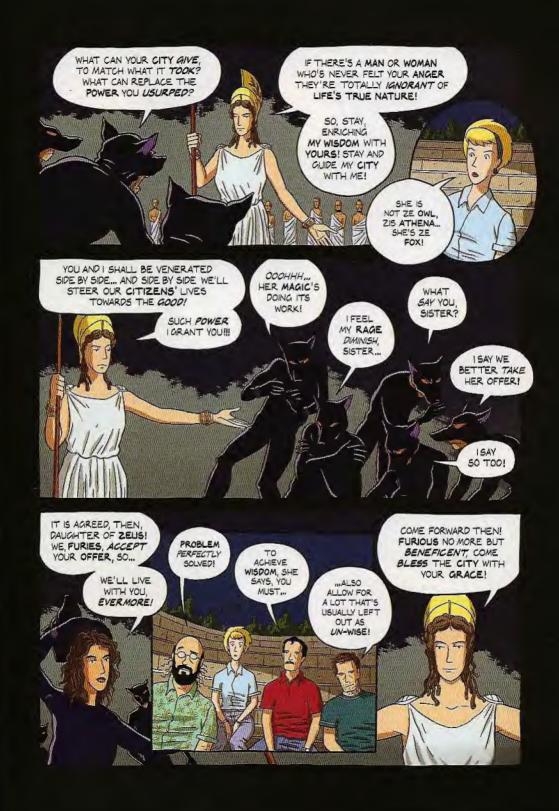
















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Logicomix and reality

Logicomix was inspired by the story of the quest for the foundations of mathematics, whose most intense phase lasted from the last decades of the 19^{th} century to the eruption of the Second World War. Yet, despite the fact that its characters are mostly real persons, our book is definitely not — nor does it want to be — a work of history. It is — and wants to be — a graphic novel.

Particularly in our reconstruction of Bertrand Russell's life, we've had to wander through an immense amount of material, to select, reduce, simplify, interpret and, very often, invent. Also, though our major characters are based as closely as possible on their real-life counterparts, we have on more than one occasion departed from factual detail, in order to give our narrative greater coherence and depth. Most of these deviations consist in inventing meetings for which there is no historical evidence — or even, in some cases, where there is evidence that they did not occur. But these imagined meetings are always based on the actual intellectual interaction of the thinkers involved, conducted in reality either through correspondence or publications.

A few examples of such deviations from fact: from the existing evidence, or lack thereof, it is safe to assume that Russell never met Frege or Cantor in the flesh; there are no indications that he was present in Hilbert's seminal 1900 lecture on the "Problems of Mathematics", although he was certainly in Paris a few days earlier, attending the Congress of Philosophy, where he met Peano; there is no evidence whatsoever that he was in the audience during Gödel's "incompleteness" talk — he probably wasn't and Hilbert certainly wasn't, though Von Neumann certainly was, and did say "it's all over" right after. Furthermore, Russell couldn't have visited Frege right after this talk, as the latter had been dead for six years. And although the timing of Frege's rabid anti-Semitic diatribes is incongruous in our book, it is totally true that he wrote them a few years earlier.

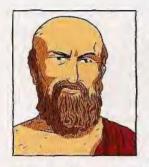
Historically keen readers can have fun locating many more such deviations from fact. For our part, we take comfort in the words of the painter Domínikos Theotokópoulos (better known as "El Greco") explaining the freedoms he took in his painting "Storm over Toledo":

I found it necessary to reduce the size of the hospital of Don Juan Tavera, not just because it covered the gate of Bisagra, but also because its dome came up too high, passing the city's skyline. And so, since I've made it smaller and moved it, I think it is better to show its façade, rather than its other sides. As for its actual position in the city, you can see it in the map.

Still, we must add this: apart from the simplification that was necessary to accommodate it into a narrative work of this kind, we have not taken any liberties with the content of the great adventure of ideas which forms our main plot, neither with its central vision, its concepts, nor — even more importantly — with the philosophical, existential and emotional struggles which are inextricably bound with it.

Notebook

The following notes are by no means necessary for the enjoyment of *Logicamix*, but may give additional information on persons and ideas. A name or term in blue indicates that it also has its own entry, while *italics*, when not used for emphasis, indicate technical terms.



Aeschylus One of the three great Greek tragedians, the precursor of Sophocles and Euripides, Aeschylus is the creator of tragedy as we know it. He introduced a second actor into the earlier dramatic form, which only used a protagonist and the chorus, thus also inventing the technique of dramatic dialogue. Born in 525 BCE in Eleusis, near Athens, he fought against the invading armies of Darius at Marathon (490 BCE) and Xerxes at Salamis (480 BCE), the latter battle

also providing the subject matter of his earliest extant play, the *Persians* (first produced in 472 BCE). The titles of seventy-nine of his plays are known to us, but only seven of these have survived in their totality, three of which constitute the *Oresteia* trilogy.



Algorithm A methodical, step-by-step procedure described in terms of totally unambiguous instructions, which starts at a specified initial condition and eventually terminates with the desired outcome. Though there is no reason why a well-written cooking recipe, or the instructions for finding a certain geographical location or address cannot be called algorithms, the term originated in mathematics, where it is still mostly used. The word "algorithm" comes from a European transcription of the name of the 9th century astronomer and mathematician Al Khwarizmi of Baghdad, who catalogued and championed these methods, having invented many of them. His compendium

of algorithms, the *Hisab al-jabr w'al-muqabala*, is generally considered to be the first algebraic treatise, the very words *al-jabr* in it also providing the root for our word "algebra". An example of a simple mathematical algorithm is the method we learn in elementary school for adding two integers: "write the two numbers one under the other with their rightmost digits justified to the right; add their last digits; if the sum is less than 10, write that number right under the other two; if it is greater than 10, write the second digit of the sum right under the other two, and add the first digit to the sum of the digits immediately

to the left ..." and so on. Probably the earliest sophisticated Western algorithm is the one given in Euclid's *Elements* for computing the greatest common divisor of two non-negative numbers. Algorithms gained prominence in the West in the 15th century with the introduction of the decimal system, which, in stark contrast with the Roman numerical system, was amenable to fast calculations, such as the one described above. Numerical algorithms played a central part in the scientific and technological revolutions. Today, algorithms are usually coded in advanced notations called *programming languages*. They are often transmitted over the Internet, and constitute the *software* that is the workhorse, platform, and backbone of computers and the Internet.



Aristotle Born in 384 BCE, in Stageira, Chalcidice, Aristotle is, with Plato, the most influential of Greek philosophers. After he left Plato's Academy, Aristotle developed his own philosophy, which departed from his teacher's in its emphasis on the systematic observation of reality and the attempt to shape general, inductive laws. Perhaps his most lasting contribution is the systematization and exposition of logic in a series of works which later commentators edited collectively

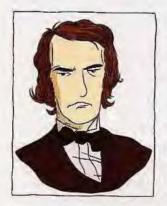
as the Organon ("instrument" or "tool"). The books comprising the Organon, i.e. The Categories, On Interpretation, The Prior Analytics, The Posterior Analytics, The Topics and the Sophistical Refutations formed the core of the canon of the study of logic until the 19th century. At the heart of Aristotle's logic is the combination of non-ambiguous statements in syllogisms to create new statements, different from the original but following necessarily from them. Aristotle also had a huge and lasting influence on mathematics, mainly through his emphasis on the notion of first principles from which any logical investigation must begin. It was this notion that found its mathematical incarnation in Euclid's concept of the axioms from which every theory has to begin. Aristotle died in 322 BCE.



Athena The ancient Greek goddess of wisdom, as well as of the arts and the city. Athena sprang in full armour from the head of Zeus, father of the Gods, whose favourite child she became. Athena was the patron goddess of ancient Athens and greatly beloved of the Athenians, to whom, according to legend, she gave the gift of the olive tree. The Parthenon, in the centre of the Acropolis,

is a temple to her — the word comes from parthenos, meaning "virgin". Atheno's role in Aeschylus' trilogy, the *Oresteia*, gives her a central role in the origin myth for the Athenian democratic invention of trial by jury, a system based on reason, as opposed to the older ones, where juridical authority emanated from a ruler's absolute power.

Axiom Since the time of Euclid, who was working in the wake of Aristotle's philosophy of logic, mathematicians agree that a workable theory must rest on some (few) agreed-upon first principles that don't require proof. This is a logical necessity if one wants to avoid, on the one hand, infinite regression (endlessly having to base something on something else) and, on the other, circuitous thinking (constructing proofs for statements which, however indirectly, assume the original statement to be true in the first place). Up to the 19th century, axioms were generally considered to be self-evident truths about the world, a view more or less still valid in Frege's idea of axioms as the reflection of an ulterior logical reality. After Hilbert, however, and under the influence of the mathematico-philosophical school of formalism, which developed from his ideas, axioms came to be seen as existing independently of any outside reality, the only requirements of an axiomatic system being: for the individual axioms their grammatical correctness (in other words, their being well-formed according to the rules of the logical language in which they are expressed), and independence (their not being derivable from the other axioms of the particular theory); and, for the whole set of axioms, its internal consistency (not containing axioms which contradict one another).



Boole, George Born in 1815, Boole was a largely self-taught mathematician who later became a professor of mathematics and logic at Queen's College in Cork, Ireland. His great contribution to mathematics is in the field of logic. In his book An Investigation of the Laws of Thought, Boole developed the idea that logical propositions can be expressed in a purely symbolic language which allows them to be manipulated by operations, similar to the operations of elementary arithmetic. At the heart of Boole's work is the idea

of a propositional calculus, constructed somewhat as Leibniz imagined it. The "Boolean search" on the Internet, involving use of the logical connectives "and",

"or" and "not", can be traced directly back to his ideas. Yet, despite the great value of his work in mathematizing logical arguments, Boole did not offer any great insights into the study of logic itself, having worked wholly within Aristotle's classical model. In Boole's system, symbols such as X and Y (essentially they are variables that can take only the two values 0 and 1) are joined via the three connectives mentioned above, as well as the "implies" connective envisaged by Aristotle. (Interestingly, the Stoic Chrysippus had already identified these connectives in the 3rd century BCE.) The application of algebraic identities, such as the three below, allow a logician to simplify logical expressions and deduce useful conclusions from them:

(X or Y) = (Y or X) not (not X) = X not (X and Y) = (not X) or (not Y)

What this logical formalism is lacking is the ability to express semantic connections between propositions. So, for example, there is no way to denote in the above that X and Y may stand for the two propositions "Plato is older than Socrates" and "Socrates is older than Plato."

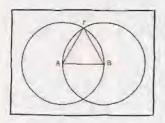
This weakness is remedied in the predicate calculus. Boole died in 1864.



Cantor, Georg Born in 1845, Cantor studied under some of the greatest mathematicians of his time, including Richard Dedekind and Karl Weierstrass. He spent the greatest part of his career teaching at the University of Halle, where he wrote his seminal papers demonstrating the great power of the ideas of set theory. His most famous theorem is that the set of so-called real numbers (all the numbers on the number line, i.e. the natural numbers 1, 2, 3... etc., together with the decimals, including 0 and the negatives) is uncountable, in other words cannot be put into a one-to-one

correspondence with the whole numbers 1, 2, 3,... etc. On the contrary, as Cantor had already proved, the set of all rational numbers, i.e. all fractions of natural numbers, such as 2/3 or 11/476, is countable and can be put in such a correspondence. As both countable and non-countable sets have an infinity of elements, Cantor's results essentially proved that there are various, mutually exclusive kinds of infinity. As his theorems were extremely counter-intuitive and thus totally unexpected, they created much skepticism about set theory in the mathematical community.

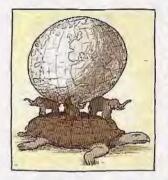
One of Cantor's teachers, the great mathematician Leopold Kronecker, as well as the mathematical giant Henri Poincaré were stronally critical of sets, though the other mathematical giant of that time, David Hilbert, was one of Cantor's greatest supporters. The identification of two distinct 'sizes' of infinity in the set of real numbers, a smaller and a larger one, ushered in the question of whether there could exist a third kind: could there be a subset of the real numbers that is neither countable nor can be put in one-to-one correspondence with the reals? Cantor conjectured that none exists, a guess ever since called "the Continuum Hypothesis" — the Continuum being another name for the number line. Cantor worked towards a proof of the Continuum Hypothesis for many years, but never achieved it. In 1940, Kurt Gödel proved that the Continuum Hypothesis is consistent with the prevailing axiomatic system of set theory (which does not amount to a proof of it). In 1963, the young American mathematician Paul Cohen proved that it is independent of it, i.e. that no real proof of the Hypothesis can be established from it, or, alternatively, that the axioms of set theory are consistent with the Hypothesis being either true or false. This discovery earned Cohen a Fields Medal, a distinction often called "the Nobel Prize of mathematics". Cantor suffered from severe emotional problems and was repeatedly hospitalized with a diagnosis of melancholia, which certain historians of mathematics have ascribed to the hostile reactions of some mathematicians to set theory, and others to the constant anxiety resulting from his fruitless attempt to prove the Continuum Hypothesis. In the last decades of his life Cantor did no mathematical work, but wrote extensively trying to substantiate two strange theories: a) that the plays of Shakespeare were in fact written by the Elizabethan philosopher Sir Francis Bacon, and b) that Christ was the natural son of Joseph of Arimathea. The second of these is a basic component of many variations of the Holy Grail legend, and a standard part of esoteric lore. Cantor died in a mental asylum, where he had been interned against his will, in 1918.



Euclid Born around 325 BCE, Euclid is the earliest Greek mathematician whose work is extant in the form in which he actually gave it — theorems of earlier mathematicians survive only as transcribed by others. He lived and worked in Alexandria, where he was associated with the Great Library. His opus magnum, the Elements, has been a best-seller for

twenty-three centuries, and is the book with the most editions in the Western world, after the Bible. Though many of the theorems appearing in it are probably not Euclid's own discoveries, the work of compilation, classification

and presentation of the existing mathematics of his day is totally his own. The *Elements* is a majestic conceptual edifice which, inspired by Aristotle and his work on logic, starts from definitions and first principles, the axioms (aitêmata — literally "requests" — in Euclid's original Greek) and then proceeds to arrive at all the theorems through rigorous proof. Though later students of logic, especially at the time of the quest for the foundations of mathematics and after, have criticized Euclid for relying too much on geometric insight or taking many more things for granted than his axioms, the influence of the *Elements* has been colossal, and it is rightly considered to be the fountainhead of the mathematical method. Euclid died around 265 BCE.



Foundations of Mathematics Since the time of Pythagoras, mathematicians have wondered about the nature of mathematical truth, the ontology of mathematical entities and the reasons for the validity of proof and, more generally, mathematical knowledge. From the Enlightenment until the middle of the 19th century, the prevailing scientific ideology saw mathematics as the only way of reaching a truth that is final, absolute and totally independent of the human mind's capacity

to understand it. The basic notions of mathematics were thought to reflect essential properties of the cosmos and the theorems to be the truths of a higher reality. This absolute faith in mathematics is reflected in the crowning of the discipline as the "Queen of the Sciences", a title whose previous holder, significantly, was theology. This view is usually termed mathematical Platonism, having its roots in the views of Plato — and, at least partly, Pythagoras before him — on the transcendent Ideas (eidê). Yet, in the 19th century this traditional belief was undermined in the minds of some people and eventually led to a serious foundational crisis in mathematics. The first of the discoveries which caused this loss of faith, dating from the time of the Renaissance, was that of the imaginary numbers (i.e. those involving the square root of minus one). But in the 19th century the appearance of non-Euclidean geometries strengthened the arguments against the "self-evident" truth of the axioms. The most troublesome of all mathematical concepts, though, was that of infinity. Problems concerning the mathematical handling of the infinite had first been alluded to by Zeno, in his paradoxes, resurfaced with the invention of the calculus in the 18th century and the counterintuitive and ill-defined concept of an infinitesimal, and peaked in the last two decades of the 19th century, most especially with set theory and Georg Cantor's

results on infinite sets. The problems that came to the surface via set theory — chief among them Russell's Paradox — culminated in severe doubts about "self-evident" truths and thus, indirectly, about the value of all mathematical knowledge. It was principally the wish to overcome these doubts that fuelled the quest for secure foundations. The "Program" proclaimed by David Hilbert in the early 1920's bearing his name, expresses the most optimistic version of the foundational dream: the creation of a formal system for all mathematics, also containing a proof that this axiomatization is consistent (i.e. can lead to no contradictions), complete (leaves no unprovable truths) and decidable (one is able to decide in every occasion whether a formula follows from the axioms or not, through the application of a set of algorithms.)



Frege, Gottlob Born in 1848, Frege spent the greatest part of his mathematical life as a profesor at the University of Jena. He is generally considered to be the father of modern logic, whose notation and method he expounded first in his Begriffsschrift (which literally translates from the German as "concept script"), published in 1879. In it, Frege departed from the earlier logicians working in the wake of Aristotle, by explicitly introducing the notion of variable in logical statements. In the place of the older type of statements like "Socrates is a man", he introduced propositions like "x is a man", propositions that can

be true or false according to the value given to x — this particular one, for example, is true if x is equal to "Alecos" but false if it's "Manga". Frege also invented the notion of *quantifiers*, the *universal* (written ∀) which makes a statement true "for every x"; and the *existential* (written ∃) which says that "there exists an x" which makes a statement true. He later applied his new logical system to the quest for the foundations of mathematics. His *Grundgesetze der Arithmetik* (*The Basic Laws of Arithmetic*) is the first great work of the school of *logicism*, whose central tenet is that mathematics is essentially a branch of logic. The first volume of the *Grundgesetze* was published in 1893 and the second, containing the addendum on Russell's Paradox, in 1903. Though Frege's logical symbolism has been abandoned as particularly cumbersome, most of the basic concepts and methods he invented still form the backbone of logic. After the *Grundgesetze*, Frege didn't do any important foundational work. In the last decades of his life he became increasingly paranoid, writing

a series of rabid treatises attacking parliamentary democracy, labour unions, foreigners and, especially, the Jews, even suggesting "final solutions" to the "Jewish problem". He died in 1925.



Gödel, Kurt He was born in 1906 in the town of Brünn, Moravia, then a part of the Austro-Hungarian Empire (the city now called Brno, in the Czech Republic). Gödel studied mathematics in Vienna, where he became fascinated with mathematical logic and the question of the foundations of mathematics. In his doctoral dissertation, he advanced Hilbert's Program by proving his Completeness Theorem, a result establishing that all valid statements in Frege's first-order logic can be proved from a set of simple axioms. In 1931, however, he proved the Incompleteness Theorem for second-order

logic, i.e. for a logic powerful enough to support arithmetic and equally or more complex mathematical theories. Gödel became one of the youngest members of the Vienna Circle, though his deeply-ingrained, idealist belief in the independent, Platonic existence of mathematical reality eventually alienated him from the other members, who embraced a materialistempirical worldview. During the late thirties, Gödel was hospitalized twice for severe melancholia. In 1940, after the Anschluss, i.e. the annexation of Austria to Nazi Germany, he managed to escape the country with his wife and took the trans-Siberian route to the United States. He became one of the first members of the Institute for Advanced Study at Princeton, where he spent the rest of his life. His most important mathematical result from this period is the proof that Cantor's Continuum Hypothesis is consistent with the axioms of set theory (i.e. that it would not be in contradiction with them, if true). At Princeton, Gödel developed a close friendship with Albert Einstein and worked for a while on the theory of relativity, establishing the mathematical possibility of a non-expanding, rotating universe, in which time travel can be a physical reality. In later life, Gödel became increasingly paranoid. He died in January 1978, at the Princeton hospital, where he had been admitted for the treatment of a non-life-threatening urinary tract problem. The cause of his death was malnutrition: he refused to eat for fear that the hospital staff was attempting to poison him.



Hilbert, David Hilbert was born in 1862 in Königsberg, Prussia (now Kaliningrad, Russia) and spent the greatest part of his life at the University of Göttingen, the world's most renowned mathematical centre at that time. He is one of the greatest mathematicians in history and, with Henri Poincaré, the greatest of his era. He made important contributions to many branches of mathematics including

invariant theory, algebraic number theory, functional analysis, the calculus of variations, the theory of differential equations and more, also pioneering new methods of proof. In 1899 he published Grundlagen der Geometrie (Foundations of Geometry), a book which gave geometry a firm basis, with new axioms, therein improving on the work of Euclid. In his famous 1900 talk at the International Congress of Mathematicians, in Paris, he attempted to give a bird's-eye view of the mathematics of the twentieth century, by way of twenty-three great open questions. Of these, now renowned "Hilbert's problems", eleven have been fully solved, seven partly, while the rest — the Eighth, also known as "the Riemann Hypothesis" is the most famous of these — are still unsolved. The Second Problem is the one demanding a proof of the consistency (the completeness was considered more or less obvious) of arithmetic — and it was this that spurred on a lot of the work on the foundations and logical structure of arithmetic, including Gödel's. In the 1920s, his ideas of the previous decades related to the foundations of mathematics culminated in what became known as "Hilbert's Program", i.e. a project to formalize all mathematics on an axiomatic basis, including a proof that this axiomatization is consistent. Hilbert's battle cries of "in mathematics there is no ignorabimus" (i.e. no "we shall not know") and "we must know, we shall know" — the latter spoken only a few days before Gödel's first announcement of his Incompleteness Theorem - encapsulate the quintessence of foundational optimism. Though the results of Gödel, Alan Turing and Alonzo Church put an end to Hilbert's grand ambition, the Program continued to exert a great influence on logic and foundational matters, and especially the development of proof theory. Though in outward appearance and behaviour Hilbert gave the impression of a paragon of normality and mental health, the way he treated his only son, Franz, raises questions. When the boy was diagnosed with schizophrenia, at age 15, his father sent him off to an asylum, where he spent the rest of his life. Hilbert never visited his son. He died in 1943.

Incompleteness Theorem In 1931, the 25 year-old Kurt Gödel proved two theorems that are sometimes referred to as "the" Incompleteness Theorem — though occasionally this form is used to denote the first of these. The completeness of a logical system is the property that every well-formed (i.e. grammatically correct by the rules of the system) proposition in it can be proved or disproved from the system's axioms. Gödel's earlier Completeness Theorem shows that there is a simple such axiomatic system for first-order logic. However, the holy grail of Hilbert's Program was to create a complete and consistent axiomatic system that can support arithmetic, i.e. the mathematics of whole numbers. Such a system would require second-order logic, i.e. a system that is also able to accept sets as values of variables. Gödel shocked the mathematical world by proving, in his famous paper "On Undecidable Propositions in the Principia Mathematica and Related Systems", that any consistent axiomatic system for arithmetic, in the form developed in the Principia, must of necessity be incomplete. More precisely, the first of the two Incompleteness Theorems establishes that in a logical axiomatic system rich enough to describe properties of the whole numbers and ordinary arithmetic operations, there will always be propositions that are grammatically correct by the rules of the system, and moreover true, but cannot be proven within the system. The second Incompleteness Theorem states that if such a system were to prove its own consistency it would be inconsistent. This was a new, devastating blow to Hilbert's Program, with its goal that a strong axiomatic system should be equipped with a proof of its own consistency.



Intuitionism This is the philosophy of mathematics created by the great Dutch mathematician Luitzen Egbertus Jan Brouwer (1881-1966), though some consider Henri Poincaré, with his strong belief in the role of intuition in mathematics, a clear precursor. Intuitionism is based on the belief that intuition and time are fundamental to mathematics, which cannot be made a-temporal or formal in the sense of Hilbert. Contrary to what logicists like Frege and Russell thought, Brouwer was convinced that logic is founded upon mathematics rather than the other way round. Also, he was totally

against the theorems of Georg Cantor in the theory of sets, considering

them ill-formed. Time-hallowed logical laws, such as that of the excluded middle, and mathematical techniques in use since the time of the ancient Greeks, such as the reductio ad absurdum, were put on trial and their use condemned. In fact, Brower believed that all the theorems making use of these in their proofs — where infinite sets of mathematical objects were concerned — should be excised from the body of mathematics, a view that made the brilliant British logician and mathematician Frank Ramsey call intuitionism "mathematical Bolshevism". Although his logic and mathematics were formalized by his student Arend Heyting, Browwer remained skeptical towards any such attempt to the end of his life.



Leibniz, Gottfried This great German philosopher, mathematician, scientist and student of logic was born in 1646. He served in the courts of several German rulers as diplomat, political advisor and historian, all the while pursuing his theoretical studies. He invented the infinitesimal calculus concurrently with, but independently from, Isaac Newton,

also proposing the notation for its operations that is still in use today. He was a strong proponent of philosophical optimism, with his theory that our world is the "best of all possible worlds", having been created by a God who is both loving and almighty. He is considered the most important logician after Aristotle and before Boole, having envisioned the calculus ratiocinator. This was a kind of computational propositional logic that would enable completely rigorous and rational decision-making which could eliminate all disagreement among rational (as Leibniz thought them) human beings. Sadly, Leibniz did not manage to realize this most coveted of his many projects. He died in 1716.

Logic The term covers a broad spectrum of disciplines — not unexpectedly, as it derives from one of the semantically richest Greek words, logos, some of whose meanings are word, speech, thought, reason, ratio, rationality, and/or concept — but can perhaps be best described as the study of methodical thinking, deduction and demonstration. The books of Aristotle's Organon present an extensive study of the deductive patterns called syllogisms, which for over two millennia were considered practically synonymous with logical thinking. Until the middle of the 19th century, logic was considered a branch of philosophy. But with the advent of Boole and his

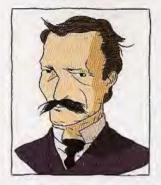
algebra of propositions and, more importantly, Frege and his "concept script" which led to a predicate calculus, it increasingly came within the province of mathematics. The new logic revealed both the underlying mathematical nature of the subject and its potential role in the creation of solid foundations of mathematics. The basic claim of the school in the philosophy of mathematics known as logicism — the school founded by Frege, of which Bertrand Russell was one of the primary exponents — was that all of mathematics can be reduced to logic or, in other words, that mathematics is essentially a branch of logic. After the years of the foundational quest, however, and especially after Gödel's results, logic became a well-developed, diversified field in the interface between philosophy and mathematics. In the second half of the 20th century it also found unexpected applications in computer science, where it provides solid foundations for the design and verification of software and hardware, as well as for databases and artificial intelligence.



Oresteia Written by Aeschylus and first performed in the theatre of Dionysus, in Athens, two years before the poet's death, in 458 BCE, it is the only extant trilogy of Greek dramas — although the satirical play Proteus, intended to be performed after the trilogy, is missing. In the trilogy's first play, the Agamemnon, the eponymous hero and leader of the Greek forces in Troy returns a victor to his hometown

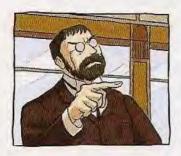
of Argos, with the captive prophetess, Cassandra. Though his wife, Clytemnestra, at first appears to rejoice at his return, she has other plans. She and her lover, Agamemnon's cousin Aegisthus, murder Agamemnon and become the new sovereigns of Argos. In the Libation Bearers, the second play, the chorus of women accompanies Agamemnon's daughter Electra to her father's tomb. The forlorn Electra is hoping for revenge, which she can only carry out with the help of her brother, Orestes, who is in exile. When Orestes clandestinely returns to Argos, he and Electra plan and execute the murder of Aegisthus and then, in a highly dramatic scene in which Clytemnestra bares her breasts before his naked sword, Orestes also kills her, his own mother. The third play, the Eumenides, or "beneficent ones", is one of the most unusual in the history of drama: all its speaking parts, apart from that of Orestes himself, are taken up by gods or other supernatural entities. The chorus consists of the Erinyes or Furies, archaic goddesses of revenge, who chase Orestes from the temple at Delphi, where

he has been ritually purified by the god Apollo, to Athens. In a totally unprecedented move — for a god anyway — Athena, the patron god of Athens, decides to let the citizens of Athens judge Orestes' case, thus giving a mythological origin—story for the democratic innovation of a court of law, with citizen jury. The trial and its aftermath develop as shown in our book's finale, though our text is only an approximate translation, slightly adapted, of Aeschylus' original words.



Peano, Giuseppe Born in 1858, this great Italian mathematician and logician spent the greatest part of his creative life as a professor at the University of Turin. Though his ideas were not as influential as Frege's in the search for the foundations of mathematics, Peano, like Frege, created a notation for first-order logic and a system of axioms for arithmetic, that is still in use — in fact, our arithmetic is formally called Peano arithmetic. He influenced Bertrand Russell

greatly, especially with his logical notation, which was much more user-friendly than Frege's. Peano believed that all mathematics could be formalized and expressed in a common, minimal language that originates from just a few axioms. But when he tried to present his own version of this universal mathematics in textbook form and use it for teaching, his students revolted, eventually causing the book's withdrawal. Inspired by his attempts to unify all mathematics by use of a common logical language, Peano later created an international auxiliary natural language, for use among people of different linguistic backgrounds, based on a simplified form of Latin which he called *Latino sine flexione*. However, like so many other artificial international languages, such as *Esperanto*, *Volapük*, *Ido* — all of them the offspring of an overoptimistic age — Peano's brainchild proved to be a mere pipe dream. Peano died in 1932.



Poincaré, Henri Born in 1854 in Nancy, France. Although he studied engineering at the École Polytechnique and the École de Mines, Poincaré was to become, with David Hilbert, the greatest mathematician of his time. He has been called the "last universal mathematician", i.e. the last one to have profound knowledge of all the mathematics

of his time. He made important contributions to many diverse fields of mathematics, among them differential equations, automorphic functions, the theory of several complex variables, probability and statistics. With his Analysis situs he essentially created the major 20th field of algebraic topology, and his work on the 3-body problem laid the groundwork for what is now called chaos theory. Despite his many great innovations, Poincaré was an extremely practical man, involved to the end of his life — alongside of his mathematical research — with the most down-to-earth of affairs, as for example the inspection of mines and an engineering project to make the Eiffel tower function as a huge antenna broadcasting time signals to navigators. He was probably the last of the great mathematicians to adhere to an older conception of mathematics, which championed a romantic faith in intuition over rigour and formalism. This stance was made famous by his reaction to the set theory of Georg Cantor as a "disease, from which mathematics will eventually be cured." His views on mathematical creation, encapsulated in his saying that "logic is barren, unless fertilized by intuition," are seen by many as the precursor of Luitzen Brouwer's school of intuitionism, a theory at the antipodes of Hilbert's strict formalism. Poincaré died in 1912.

Predicate calculus Often used synonymously with predicate logic and first-order logic, the predicate calculus is Frege's extension of the propositional logic developed by Boole. In the predicate calculus, elementary propositions (or predicates) are composite objects of the form P(a, b, c,...), where P is a symbol in the language, and a, b, c, etc. are constants or variables. For example, if "older" is a propositional symbol, "Plato" is a constant and "x" is a variable, then "older (Plato, x)" is a well-formed proposition, describing that Plato is older than x. Propositions of this type can then be combined by Boole's connectives "and", "or", "not" and "implies" and prefixed by Frege's quantifiers, such as "for all x" (written \forall) and "there exists y" (written 3). Thus, "there exists x older(x, Plato)" means that there is (at least) one individual who is older than Plato. Evidently, this is a much more ambitious attempt at creating Leibniz's calculus ratiocinator than Boole's simpler formal logic. By employing symbols from various fields of mathematics (such as "<", "+", and so on) one can create predicates expressing mathematical statements in this formal, logically exact language. For example, the theorem in arithmetic stating that every integer is either odd or even can be written thus:

 $(1+y+y=x \text{ or } y+y=x) y \in x \forall$

Rigorously defined, the version of the predicate calculus called first-order logic employs simple mathematical objects as variables, whereas in second-order logic variables can also be sets, making possible statements like "there is a set S". This, more powerful language, can express all known mathematics. Whether a sentence in the predicate calculus, first- or second-order, is true or false depends on the model whereby the sentence is interpreted. Thus, for example, the simple arithmetical theorem given above is true of the whole numbers in the ordinary interpretation of "+", but becomes false if we interpret the symbol "+" as multiplication. However, some sentences — called valid — are true independently of interpretation, because they embody basic properties of Boolean connectives and quantifiers. Kurt Gödel's Completeness Theorem provides a simple, complete axiomatic system for proving validity in first-order logic.

Principia Mathematica The extremely influential, but highly controversial, essentially unfinished work in which Alfred North Whitehead and Bertrand Russell attempted to rescue Frege's grand project to create foundations of mathematics built on logic, in the wake of the crisis brought on by Russell's Paradox. The title Principia Mathematica (i.e. "Principles of Mathematics") in itself provoked controversy, as it is the exact same as that of Newton's greatest work; many in the British mathematical community thought this choice to be in bad taste, if not actually blasphemous. The three volumes of the Principia, published in 1910, 1912 and 1913, were based on a developed version of Russell's theory of types, the so-called "ramified", which imposed a hierarchical structure on the objects of set theory. This could not be made to yield the required results, however, without the addition of what Russell called an axiom of reducibility, which eventually became one of the main reasons for negative criticism of the whole work. Logicians found this axiom extremely counter-intuitive, a far-fetched and basically artificial method to sweep the very problem it was trying to solve under the rug. Despite the fact that the Principia fell short of its authors' immense ambition, it had a huge influence on the shaping of modern logic, its greatest effect possibly being the inspiration and context it provided Kurt Gödel for his groundbreaking discovery, the Incompleteness Theorem.



Proof The process of arriving at the logical verification of a mathematical or logical statement. starting from a set of agreed-upon first principles (these could be either axioms or already proven statements, deriving from these axioms), and proceeding by totally unambiguous and unabridged logical steps or rules of inference. The demonstrations of geometric propositions in Euclid's Elements were considered for over two millennia to set the standard of excellence to which mathematical proof should aspire. Yet, towards the end of the 19th century his method came under logical and philosophical scruting and was found to lack, principally, in two directions: a) in its sense of the logical "obviousness" of the axioms, and b) in its logical gaps, where Intuition - which, in Euclid's case was mostly visualgeometric - took over from strict application

of a formal system of rules. In a sense, Frege's and Russell and Whitehead's logicist project was developed as a reaction to the imperfections found in Euclid's proofs, as well as all those developed in his wake. The logicists, as well as the formalists working on the foundations of mathematics, aimed at a fully developed theory and practice of rigorous proof, by which arithmetic (as the basis of all mathematics) would begin from a small number of consistent axioms, and eventually lead, via proof, to the full range of truth. Hilbert's seminal question, which he called the Entscheidungsproblem ("decision problem"), posed in 1928 and answered seven years later by Alan Turing, is equivalent to the demand for a totally powerful apparatus of proof, which can provide a provable or unprovable response to any mathematical statement by virtue of a rigorous algorithm.



Russell, Bertrand Born in Wales, in 1872, Bertrand Arthur William, the Third Earl Russell — this is his full name, by virtue of his noble descent — was the grandson of the important politician Lord John Russell, whose title he eventually inherited. An orphan at the age of four, Russell was raised by his paternal grandparents, and after his grandfather's death two years later, exclusively by his grandmother, Lady Russell. He grew up at the family home of

Pembroke Lodge, in Richmond Park, to the west of London. Russell is now perhaps best known to a wider public for his work in philosophical exposition. His History of Western Philosophy, published in 1945, remains to this day a classic of idiosyncratic, yet intelligent and highly readable exposition of complex ideas. And while his later work as a pro-peace and anti-nuclear activist also earned him international fame, Russell's areatest contribution is in mathematical logic, ranking him, along with Aristotle, Boole, Frege and Gödel, with history's greatest logicians. Despite the momentous importance of his work in the establishment of a scientific logic, its direct influence on Gödel's great discoveries, and the indirect on the Vienna Circle's "scientific worldview" and the philosophies of logical positivism and logical empiricism, Russell's work in logic essentially ends with the Principia Mathematica, the book he co-authored with Alfred North Whitehead, completed just before he turned forty. Russell considered the Principia essentially a failure, as it fell short of his —and the other logicists'— grand ambition, of founding mathematics securely on logic. Russell married four times and fathered three children. His first son, John, as well as John's daughter, were diagnosed as schizophrenics, and the latter committed suicide. This pathology was very possibly another instance of the streak of mental illness running in the family, manifest both in Russell's uncle William and his aunt Agatha. During the last decades of his life, Russell gave all his energy to the struggle for nuclear disarmament, becoming an emblematic figure of pacifism. He died in 1970.



Russell's Paradox Discovered in 1901, as Russell was working on his first book on the foundations of mathematics, the Principles of Mathematics (published in 1903), the Paradox, in the form originally expressed, shows an essential flaw in Cantor's set theory, developed from Bolzano's simple concept of a "collection of elements with a common property". By the generality of this definition, which Frege extended to the realm of logic, one can speak

of a "sets of sets" and thus, eventually of the "set of all sets".

Of the elements of this all-encompassing set one defines the property of "self-inclusiveness", i.e. of a set containing itself as an element. Thus, for example, the set of all sets is a set (and thus contained in itself), as is the set of all entries in a list (it can appear as an entry in a list),

but the set of all numbers is not a number and thus not contained in itself. By virtue of this property, we can define the "set of all sets which don't contain themselves", and ask, with the young Russell, the avestion: "Does this set contain itself or not?" See what happens: if it does contain itself, it follows that it is one of the sets which don't contain themselves (as this is the property that characterizes elements of this set) and thus cannot contain itself. But if it doesn't contain itself, then it does not have the property of not containing itself, and thus does contain itself. This situation, in which assuming something implies its negation, and vice versa, is called a paradox. When a paradox, such as Russell's, arises in a theory, it is a sign that one of its basic premises, definitions or axioms is faulty. Though historically developed within the context of the theory of sets, Russell himself later viewed his paradox as essentially having to do with self-reference, i.e. with statements referring to themselves, such as Euboulides' "I am now lying to you."



Self-reference Literally, the quality of a statement of referring to itself. However, it is also used more generally in logic to characterize statements which include themselves within their scope of reference, as in the "barber" story used to explain Russell's Paradox. The barber lives in a town wherein a law decrees that

"all residents of the town must either shave themselves or be shaved by the barber." This law is self-referential as the barber, apart from being "the barber" referred to, is also one of the "residents of the town". Self-reference has played a seminal role in logic and mathematics, already from the time of the Greeks. From Euboulides' self-referential statements, to Cantor, whose proof of the non-denumerability of the real numbers relies heavily on a numerical variant of self-reference, to Russell and his paradox, and to Gödel. In fact, Gödel proved his Incompleteness Theorem by creating, in the context of modern logic, a statement that is quite similar in spirit to that of Euboulides, with one crucial difference: while Euboulides states "this statement is false", Gödel's ingenious variant essentially says, in the language of arithmetic, "this statement is unprovable." Any consistent axiomatic theory in which one can formulate such a statement must be necessarily incomplete: for either this statement is false, in which case it is both false and provable,

contradicting the *consistency* of the axiomatic system, or *true*, in which case it is both true *and* unprovable, establishing its *incompleteness*.

Set theory The study of collections of objects united by a common property - in some cases this property can be nothing more than the fact that they are defined to be members of the same set, as for example in the arbitrarily defined set whose elements are the numbers 2, 3, 8, 134, 579. Sets were first studied by the Czech mathematician Bernard Bolzano (1781-1848), who also introduced the term Menge ('set') and defined the notion of a set's cardinality, i.e. of its "size" in a way not directly involving measurement. Thus, one can speak of two sets having the same cardinality if their elements can be put in a one-to-one correspondence - without ever needing to know via the precise number of these elements. This has the great advantage that it also works for infinite sets, where the notion of number does not apply: mathematicians don't think of "infinity" as a number. However, some seeming paradoxes, such as the fact that the whole and the even numbers can be put into a one-to-one correspondence (just by multiplying each whole number by 2, or dividing each even number by 2), thus showing a subset to have the same cardinality as the containing set, prevented Bolzano from developing the theory further. The advanced mathematical discipline of set theory was arguably born on December 7, 1873, when Georg Cantor wrote to his teacher. Richard Dedekind describing his proof of the non-denumerability of the real numbers (the set of the whole numbers, decimals, zero and the negative numbers), as opposed to the denumerability of the rationals (all fractions), which Cantor also proved — denumerability is defined as a one-to-one correspondence with the natural numbers 1, 2, 3... etc. The concept of a set is almost too primitive to merit a mathematical definition, and is practically impossible to define informally without the use of some synonym (here we used the word "collection"). It is precisely this "naturalness" of the concept in Bolzano's and Cantor's work that led to Russell's Paradox. To overcome it, and to rule out the flawed concept of "the set of all sets" it allowed for, one has to come up with bottom-up constructions and axioms for sets, as in the Principia Mathematica and, later, the system called "ZFC", from the names of its two creators, Ernst Zermelo and Abraham Fraenkel, and the Axiom of Choice, a necessary additional axiom that allows the theory to deal with infinite sets. Set theory is considered by some the most basic branch of mathematics, as all others can be defined in terms of it. This was the gist of an over-ambitious project undertaken, from the 1950s onwards, by the aroup of brilliant French mathematicians writing under the pen name of "Nicolas Bourbaki".

Tractatus Logico-Philosophicus Ludwig Wittgenstein wrote his seminal philosophical work during WWI, building on his pre-war notebooks and ideas on logic. It contains his solution of (in his own words) "all the problems of philosophy", dealing with the world, representation, and language. Originally called Logische-Philosophische Abhandluna ("Logical-Philosophical Treatise"), it was renamed for its English publication under the influence of G. E. Moore, with his predilection for Latin titles. In the Tractatus, Wittgenstein uses many techniques and ideas from logic, especially those of Frege and Russell, as well as insights from totally different philosophical positions, mostly that of Arthur Schopenhauer. Though publication by the then totally unknown Wittgenstein was only made possible when Russell accepted to write an introduction, calling the book "an important event in the philosophical world", the Tractatus was the cause of the two men's falling-out. Wittgenstein considered Russell's — not altogether appreciative introduction to his work to be fraught with misunderstandings and philosophical errors, while Russell saw in the Tractatus the first signs of Wittgenstein's decline — as he saw it — into mysticism. The tight structure of the book proceeds with seven main propositions, each developed in a chapter, which are further developed in propositions arranged by a rather pedantic, and often somewhat confusing, system of numbering. The first two propositions (1 and 2) expand the positions that "the world is all that is the case", and that "what is the case" are facts, and combinations of facts. This is a departure from classical philosophy and the metaphysics of Aristotle in particular, according to which the world consists of objects. In the logical language of the Tractatus, objects do figure within states of affairs, but in complex combinations and relationships with each other, and not as elementary units. The next two propositions (3 and 4) develop mostly what has been called the picture theory of language, whereby a "thought is a proposition with sense." Passing here to representation and language, Wittgenstein delimits thoughts to logical propositions, but within a context and in reference to the world. This is perhaps the most subtle part of the book, and also the one which relates to Wittgenstein's idea of mathematics and logic as machines for producing tautologies. Propositions 5 and 6 develop the idea that "propositions are truth functions of elementary propositions", in which mathematical-symbolic notation is used to explain precisely what a truth function is. Here Wittgenstein uses logic to define propositions (and thus language and thought) as the combinations of atomic, or elementary propositions, combined through Boole's laws of composition. This part of

the book actually contains the first mention of what is now known as the "truth table method" for dealing with Boolean functions. The book's final clause, proposition 7, is: "What we cannot speak of, we must pass over in silence." (This and other quotes are from the D.F. Pears and B.F. McQuinness translation.) This last proposition was given two highly divergent interpretations, the extreme positivist one of the Vienna Circle, by which what one "cannot speak of" (logically) is, quite literally, nonsense, and the one that Wittgenstein and others himself later gave, which Russell termed "mystical", according to which what "one cannot speak of" is the truly important. The Tractatus is one of the most influential and closely-studied books in Western philosophy. Its influences are legion and it may have also influenced — and certainly was vindicated by — the way in which computers and databases model the world today.



Turing, Alan Born in London in 1912, this great British mathematician is generally considered to be the father of computer science. Turing contributed to many areas of mathematics, but is mostly remembered for one of his earliest results in logic. While a student at Cambridge, he became fascinated by the foundations of mathematics and especially the Incompleteness Theorem of Kurt Gödel, which inspired him to study Hilbert's Entscheidungsproblem ("decision problem"), a question that had survived Gödel's analysis. The Entscheidungsproblem asks whether, given a logical system, there is an algorithm for deciding whether a proposition is

provable within the system or not. Turing's answer was a devastating "no". To reach this, he first had to define rigorously the notion of algorithm. His ingenious definition in terms of a theoretical "machine" with a central control and a tape for memory, input and output, anticipated in important ways the digital computer and has had, since then, an enormous influence on computational practice and thought. Turing machines — as they are now called — share with today's computers the key property of universality, in that a machine can carry out any computational task, provided it is supplied with an appropriate program for it. Two other mathematicians, Alonzo Church (later Turing's thesis advisor at Princeton) and Emil Post, came up independently, and at about the same time, with algorithm formalisms that were ultimately shown

equivalent to Turing's. Yet his formalization had the greatest impact, mainly because of the extreme simplicity of its basic construction, which can, nevertheless, achieve extremely complex results. The work of Turing - as well as that of the others mentioned - on algorithms and methods for the general solvability of problems, is an obvious outgrowth of the foundational guest and thus, in a sense, its culmination. During World War II. Turing presided over the design and construction of two series of electronic computers, the "Bombe" and the "Colossus". These were used successfully - and crucially for the war effort - for breaking several German cryptographic codes, including the notoriously hard "Enigma" of the German navy. After the war, Turing worked in the fledgling British computer industry, did important work in biology and founded the field of artificial intelligence by proposing what became known as the Turing test, a method for determining whether an artifact "can think". Always interested in sports and games — he was an accomplished long-distance runner — Turing was the first to develop ideas for a chess-playing program, making mastery in the game one of the goals towards which the designers of intelligent machines should strive. In 1952 he was prosecuted on account of his homosexuality, then a punishable offense in Britain. As an alternative to a jail sentence, he agreed to undergo an experimental "treatment" with estrogens, which probably caused the severe depression which led him to take his own life, in 1954.



Vienna Circle A group of philosophers and philosophically-minded scientists, who met in Vienna between 1924 and 1936. Their main aim was two-fold: to build a strong empiricist philosophy using the insights into scientific methodology garnered from recent advances in logic, mathematics and physics, and to apply

the methodology of the physical sciences to the social. The scientifically-trained philosopher of science Moritz Schlick is generally recognized to be the group's leader. Some of the most prominent members were: the mathematicians Hans Hahn, Olga Hahn-Neurath, Gustav Bergmann, Karl Menger, and Kurt Gödel for a short period of time; the physicist Philipp Frank; the social scientist Otto Neurath and the philosophers Viktor Kraft and Rudolf Carnap. The group met informally Thursday evenings at Vienna's "Café Central", but was later constituted as a society with public meetings. Despite the group's informal nature,

the members had a common core of philosophical beliefs, expressed in a sort of manifesto, titled "The Scientific Conception of the World". The members of the Circle declared that the work of Frege, Russell and Einstein provided their first inspiration, while the Tractatus Logico-Philosophicus of Ludwig Wittgenstein functioned as their direct model. The philosophies of logical positivism and logical empiricism. expressing the worldview of the members of the Circle, state that knowledge comes from experience — and thus, basically, from scientific observation and experiment — developed into theory through logical analysis and synthesis. Still, following the Tractatus, members of the Vienna Circle held that logic and mathematics only deal in tautologies. and thus do not provide knowledge as such, but only one of the tools for the elaboration of empirical knowledge. According to the worldview of the Circle, statements that cannot be reduced to experience (such as theological or ethical pronouncements) cannot be right or wrong, as they are - quite literally - non-sense, having no meaning. The most extreme version of this tenet, due to Carnap, actually required that for a statement to be meaningful, its truth or falsity must be verifiable by an algorithm reducing it to observable truths — a new incarnation of Leibniz's "calculemus". Carnap later tried to reconcile this view with the Incompleteness Theorem. Though the Vienna Circle, in its original form, was dissolved in 1936, after Schlick's murder by a paranoid ex-student and Nazi sympathizer, its spirit continued to live on. Most of its members managed to flee Austria and emigrate to England and the United States, where they had a major influence on the development of post-war philosophy.



Von Neumann, John Born in Budapest in 1903 ("John" is the anglicized form of the Hungarian "Janos"), von Neumann showed very early signs of unusual intellectual prowess, being able to do mental division of 8-digit numbers and converse in ancient Greek by the age of six. He studied mathematics in Budapest, obtaining a PhD at 22, meanwhile also working towards a degree in chemical engineering at the renowned Technical University of Zürich, to please his father.

He rapidly became the star mathematician of his generation, legendary for his penetrating and rapid-fire mathematical genius. Upon attending the lecture where Gödel announced the first Incompleteness Theorem,

von Neumann was the first to realize the result's import, and did indeed proclaim "it's all over" after the talk. But he made crucial suggestions to Gödel right after it, and went on to prove the second Incompleteness Theorem — which however Gödel himself had also proven independently in the meantime. Von Neumann never worked on the foundations of mathematics again. Possessing a wide-ranging mathematical genius, he made contributions to many different branches, he has been called "the last of the great mathematicians", having made great contributions to many different branches of mathematics, among them set theory, operator algebras, ergodic theory and statistics. He also did important work in quantum theory, fluid mechanics and mathematical economics, being the co-founder (with economist Oscar Morgenstern) of the field of game theory. During WWII, he was one of the brains behind the atomic bomb, and after it headed the U.S. government committee in charge of the construction of the hydrogen bomb. Perhaps most important of all his work, however, was his contribution to the creation of computers. While he was working as a consultant in the design of one of the first electronic computers, in 1946, and influenced by Alan Turing's ideas, von Neumann developed an array of fundamental design principles, postulating, among others, a central processing unit and separate memory devices where both data and programs are both stored. Practically all subsequent computer designs have been based on this basic model, now known as the von Neumann architecture. Von Neumann went on to become one of the first great computer scientists, especially excelling in what now would be called scientific computing, i.e. the use of computers for scientific research. He died of cancer — possibly the result of his attendance of thermonuclear tests - in 1957.



Whitehead, Alfred North English mathematician and philosopher. Born in 1861, he studied mathematics at Cambridge, where he also taught for many decades. In 1891 he married Evelyn Wade, an Irish woman much younger than himself. Before his intense, decade-long collaboration with Bertrand Russell on the *Principia Mathematica*, Whitehead published his book *Universal Algebra*, an attempt to study the types of symbolic reasoning

in various algebraic systems from a very modern — for its time — formal viewpoint. After Russell's abandonment of the *Principia*, in 1913, Whitehead

tried to write a fourth volume, on geometry, but never completed it. The two men had very little interaction after the publication of the *Principia*, and Whitehead did not contribute to the 1925, second edition of the book, having moved on to mathematical physics and later philosophy. He died in 1947.



Wittgenstein, Ludwig Wittgenstein is considered by many to be the greatest philosopher of the 20th century. He was one of the eight children of industrialist Karl Wittgenstein, one of Austria's wealthiest and most powerful men, and a great patron of the arts. Of his four brothers, three committed suicide in early manhood, while the fourth, Paul, went on to become a renowned concert pianist. After two years of engineering studies, Wittgenstein

developed a strong interest in logic and the foundations of mathematics. He went to see Frege, who suggested that he go to Cambridge to study with Russell, a piece of advice Wittgenstein followed. The association deeply influenced both men, but probably the teacher more than the student. During his service with the Austro-Hungarian army in WWI, Wittgenstein won several medals for his valour, his citations underlining his "sang-froid under fire". He was eventually captured by the enemy and completed his magnum opus, the Tractatus Logico-Philosophicus, in an Italian prisoners' camp. After the war he donated the huge fortune left to him by his father to his three sisters and, having, as he believed and declared, "solved all the problems of philosophy" with the Tractatus, he worked as a gardener, architect, and eventually as a teacher in a small village in Lower Austria. In 1929, possibly inspired by interactions with members of the Vienna Circle, as well as attending a lecture on the philosophy of mathematics by Luitzen Brower, on intuitionism, Wittgenstein returned to Cambridge and philosophy. He retracted his earlier work as dogmatic and went on to create a new, extremely influential philosophical stance often referred to as "the late Wittgenstein". Unlike the ideas in the Tractatus, Wittgenstein did not attempt to put his later philosophy in a systematic treatise, but presented them in a series of more or less independent remarks. Many of these he saw as forming a book, which was posthumously published as Philosophical Investigations - this. as well as a few books based on his notebooks, or transcripts of lectures or discussions, are all that we have of his later thought. This is a

philosophical position of an extreme anti-dogmatic nature, focusing on language and psychology (what we now call cognitive psychology), instead of logic and objective truth, and on fuzzy concepts such as "family resemblance" and "language games" instead of clear definitions and propositions. In this later phase, Wittgenstein's thinking is characterized by a vicious criticism of philosophy as it had been practiced until then, by others but also himself — it was for this criticism more than anything else, that Russell was dismissive of his later work, referring to Wittgenstein's decision to "become a mystic". Most of his negative criticism of mathematics — which he increasingly came to view as a purely practical activity, a craft legitimized only by its use in application — is contained in transcriptions of his lecture notes at Cambridge.

Of particular interest is the dialogue with one of the attendees at these lectures, Alan Turing, who strongly disagreed with his ideas on mathematics. Wittgenstein died in 1951.

Bibliography

In preparation for *Logicomix* we read many books — in addition to those we had read earlier, before the idea for the project was even born — and consulted many more, and even more articles. Of all these, we mention here very few, chosen either for the wealth of the information they contain, for their astuteness, profundity and/or synthetic ability. Clearly, this list represents a personal choice, and nothing more: these are the books that we most liked and found most useful.

- Andersson, Stefan. In Quest of Certainty: Bertrand Russell's Search for Certainty in Religion and Mathematics Up to the Principles of Mathematics (1903).

 Stockholm: Almqvist & Wiksell International, 1994.
- Davis, Martin. The Universal Computer: The Road from Leibniz to Turing. New York: W. W. Norton & Company, 2000.
- Gray, Jeremy J. *The Hilbert Challenge*. Oxford: Oxford University Press, 2000. Janik, Allan, and Stephen Toulmin. *Wittgenstein's Vienna*. New York: Simon and Schuster, 1973.
- Monk, Ray. Ludwig Wittgenstein: the Duty of Genius. London: Jonathan Cape, 1990.
- Bertrand Russell: the Spirit of Solitude. London: Jonathan Cape, 1996.
- Bertrand Russell: the Ghost of Madness, 1921-1970. London: Jonathan Cape, 2000.
- Reid, Constance. Hilbert. Berlin: Springer-Verlag, 1970.
- Rota, Gian-Carlo. 1997. "Fine Hall in its Golden Age". In *Indiscrete Thoughts*, ed. Fabrizio Palombi, 4-20. Boston: Birkhauser Verlag AG.
- Russell, Bertrand. My Philosophical Development. London: George Allen & Unwin, 1959.
- The Autobiography of Bertrand Russell, 3 vols. London: George Allen & Unwin, 1967–1969.
- Griffin, Nicholas, ed. The Selected Letters of Bertrand Russell. London: Routledge, 2002.
- Scharfstein, Ben-Ami. *The Philosophers*. Oxford: Oxford University Press, 1980.
- Stadler, Friedrich. The Vienna Circle, Studies in the Origins, Development, and Influence of Logical Empiricism. English translation by Camilla Nielsen. Vienna: Springer-Verlag, 2001.
- Van Heijenoort, Jean. From Frege to Gödel. Cambridge: Harvard University Press, 1967.
- Wittgenstein, Ludwig. *Tractatus Logico-Philosophicus*. (English translation: D. F. Pears and B. F. McGuinness. London: Routledge and Kegan Paul, 1961.)



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advance Praise for LOGICOMIK

"It's difficult not to be dazzled by apostolos Doxiadis and Christos Papadimitriou's Logicomix: It's a biography of the mathematician/philosopher Bertrand Russell, a fiercely engaging examination of his elusive attempt to isolate the logical foundations of mathematics, and a rousing historical yarn. and all of Logicomix's storytelling and intellectual pyrotechnics are delineated in extraordinarily crisp, cleverly designed and beautifully colored artwork by the team of alecos Papadatos and annie Di Donna. What a comic book! Pasily one of the most impressive combinations of popular art and serious history that I've encountered in prose or in comics."

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"This magnificent book is about ideas, passions, madness, and the fierce struggle between well-defined principle and the larger good. It follows the great mathematicians—Russell, Whitehead, Frege, Cantor, Hilbert—as they agonized to make the foundations of mathematics exact, consistent, and complete. and we see the band of artists and researchers—and the all-seeking dog Manga—creating, and participating in, this glorious narrative."

—BBRRY MBZUR, Gerhard Gade University Professor at Harvard University, and author of Imagining Numbers (porticularly the square root of minus fifteen)

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-MICHOEL HORRIS, professor of mathematics at Université Paris 7 and

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